(6.) (11 points) The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse"—that is, an ellipse whose axes are not parallel to the coordinate axes.

(a) Find the points at which this ellipse crosses the x-axis.

Plug in $y = 0$ and solve for x:

$$x^2 = 3$$

$$x = \sqrt{3}, -\sqrt{3}$$

(b) Show that the tangent lines at these points are parallel.

Find $\frac{dy}{dx}$, using implicit differentiation:

$$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

When $y = 0$, $x = \pm\sqrt{3}$, we have $\frac{dy}{dx} = 2$.

Since $\frac{dy}{dx}$ is the slope of the tangent line, we see that both lines have slope 2, so they are parallel.

(c) Under what conditions on x (if any) would a tangent to the curve be vertical? Explain.

The tangent line is vertical when the denominator of $\frac{dy}{dx}$ is zero (and the numerator is not zero). This happens when $2y = x$ (and $y \neq 2x$).