(7.) (10 points) For some positive constant C, a patient's temperature change, T, due to a dose, D, of a drug is given by

$$T = f(D) = \left(\frac{C}{2} - \frac{D}{3}\right) D^2$$

(a) What dosage maximizes the temperature change?

$f'(D) = CD - D^2$, $f''(D) = C - 2D$

Critical point when $f'(D) = 0$, that is, when $D = 0$ or $D = C$.

Since $f''(C) = -C < 0$, the function $f(D)$ has a maximum when $D = C$.

Note that since $D = C$ is the *only* critical point for $D > 0$, and it is a local maximum, it must also be a global maximum for $D > 0$.

(Note that $f''(0) = C > 0$, so $D = 0$ is a minimum.)

(b) The sensitivity of the body to the drug is defined as dT/dD. What dosage maximizes sensitivity?

Critical point when $(\frac{dT}{dD})' = f''(D) = 0$.

This happens when $D = C/2$. Check that $(\frac{dT}{dD})''(C/2) = f'''(C/2) = -2 < 0$, so this is indeed a maximum, and since this is the only critical point, $D = \frac{C}{2}$ is a global maximum.