(7.) (10 points) For some positive constant $C$, a patient’s temperature change, $T$, due to a dose, $D$, of a drug is given by

$$T = f(D) = \left( \frac{C}{2} - \frac{D}{3} \right) D^2$$

(a) What dosage maximizes the temperature change?

$$f'(D) = CD - D^2, \quad f''(D) = C - 2D$$

Critical point when $f'(D) = 0$, that is, when $D = 0$ or $D = C$. Since $f''(C) = -C < 0$, the function $f(D)$ has a maximum when $D = C$.

Note that since $D = C$ is the only critical point for $D > 0$, and it is a local maximum, it must also be a global maximum for $D > 0$.

(Note that $f''(0) = C > 0$, so $D = 0$ is a minimum.)

(b) The sensitivity of the body to the drug is defined as $dT/dD$. What dosage maximizes sensitivity?

Critical point when $(\frac{dT}{dD})' = f''(D) = 0$.

This happens when $D = C/2$. Check that $(\frac{dT}{dD})''(C/2) = f'''(C/2) = -2 < 0$, so this is indeed a maximum, and since this is the only critical point, $D = \frac{C}{2}$ is a global maximum.