(8.) (14 points) Ship A is travelling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is travelling due north away from Lighthouse Rock at a speed of 10 km/hr. Let \( x \) be the distance between Ship A and Lighthouse Rock at time \( t \), and let \( y \) be the distance between Ship B and Lighthouse Rock at time \( t \), as shown in the figure below.

(a) Find the distance between Ship A and Ship B when \( x = 4 \) km and \( y = 3 \) km.

\[
\text{distance} = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5 \text{ km}
\]

(b) Find the rate of change of the distance between the two ships when \( x = 4 \) km and \( y = 3 \) km.

\[
D^2 = x^2 + y^2 \quad \text{where} \quad D, \ x, \ \text{and} \ y \ \text{are all functions of} \ t.
\]

Thus, \( 2D \frac{dD}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \)

\[
\frac{dx}{dt} = -15 \text{ km/hr} \quad \text{and} \quad \frac{dy}{dt} = 10 \text{ km/hr}, \ \text{so:}
\]

\[
\frac{dD}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{D}
\]

When \( x = 4 \), \( y = 3 \), we have:

\[
\frac{dD}{dt} = \frac{(4)(-15) + (3)(10)}{5} = -6 \text{ km/hr}
\]

(c) Let \( \theta \) be the angle shown in the figure. Find the rate of change of \( \theta \) when \( x = 4 \) km and \( y = 3 \) km.

Note that \( \tan(\theta) = \frac{y}{x} \).

Thus, \( \frac{1}{\cos^2(\theta)} \left( \frac{d\theta}{dt} \right) = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2} \),

\[
\frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{16} \left( \frac{16}{25} \right) = \frac{85}{16} \left( \frac{16}{25} \right) = \frac{85}{25} = \frac{17}{5} \text{ radians/hour}.
\]