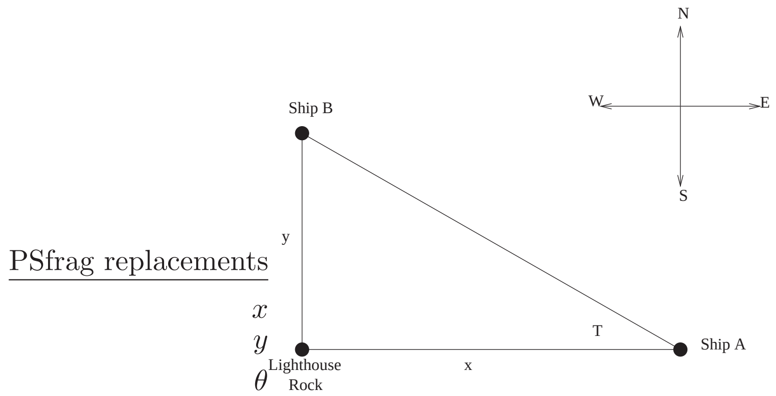
(8.) (14 points) Ship A is travelling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is travelling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure below.



(a) Find the distance between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5 \text{ km}$$

(b) Find the rate of change of the distance between the two ships when $x = 4$ km and $y = 3$ km.

$D^2 = x^2 + y^2$ where D, x, and y are all functions of t.

Thus, $2D(\frac{dD}{dt}) = 2x(\frac{dx}{dt}) + 2y(\frac{dy}{dt})$

$\frac{dx}{dt} = -15\frac{km}{hr}$, and $\frac{dy}{dt} = 10\frac{km}{hr}$, so:

$$\frac{dD}{dt} = \frac{x(\frac{dy}{dt})+y(\frac{dx}{dt})}{D}$$

When $x = 4$, $y = 3$, we have:

$$\frac{dD}{dt} = \frac{(4)(-15)+(3)(10)}{5} = -6\frac{km}{hr}$$

(c) Let θ be the angle shown in the figure. Find the rate of change of θ when $x = 4$ km and $y = 3$ km.

Note that $\tan(\theta) = \frac{y}{x}$.

Thus, $\frac{1}{\cos^2(\theta)}(\frac{d\theta}{dt}) = \frac{x(\frac{dy}{dt})-y(\frac{dx}{dt})}{x^2}$,

$$\frac{d\theta}{dt} = \frac{4(10)-3(-15)}{16}(\frac{16}{25})$$

$$= \frac{85}{16}(\frac{16}{25}) = \frac{85}{25} = \frac{17}{5}\frac{radians}{hour}.$$