3. (2+8 points) The logistic model for population growth is a model that accounts for the fact that population cannot grow indefinitely. The formula for the logistic model is given by $P(t) = \dfrac{L}{1 + Ae^{-kt}}$ where L and A are positive constants.

(a) The carrying capacity is the horizontal asymptote of $P(t)$. What is the carrying capacity? What does this mean in practical terms?

The carrying capacity $= \lim_{t \to \infty} P(t) = L$. This gives an upper bound on the population, i.e., the population can approach a value of L but can never quite reach it.

(b) List the steps you would take to find the value of t for which the population is growing the fastest? Give reasons for each step. You do **NOT** have to carry out any of these steps!!!!

1. Find P' as this gives the rate of growth and is the function we are interested in maximizing.

2. Find P'' in order to find the critical points of P'.

3. Find the critical points of P' by determining where P'' is equal to zero or undefined.

4. Test the critical points of P' by either
   - taking the third derivative and testing the critical points,
   - looking at the sign of P'' around the critical points, or
   - giving a graphical argument based on the graph of P or P'.