4. (6 points) The shape of a balloon used by a clown for making a balloon animal can be approximated by a cylinder. As the balloon is inflated, assume that the radius is increasing by 2 cm/sec and the height is given by $h = 2r$. At what rate is air being blown into the balloon at the moment when the radius is 3 cm?

The formula for the volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. We know that $h = 2r$, so we can write $V = 2\pi r^3$. Taking the derivative with respect to t of both sides we get

$$\frac{dV}{dt} = 2\pi 3r^2 \frac{dr}{dt}.$$

We are interested at the time when $r = 3$ and $\frac{dr}{dt} = 2$, so

$$\frac{dV}{dt} = 108\pi \text{cm}^3/\text{sec}.$$

5. (8 points) In introductory physics one learns the formula $F = ma$, connecting the force on an object, F, with the mass of the object and the acceleration that the object experiences under the force. One also learns the formula $p = mv$ where p is the momentum of an object, m is the mass, and v is the velocity.

(a) Derive the formula $F = ma$ given that $\frac{dp}{dt} = F$, assuming that the mass is constant and that $p = mv$. Explain your answer.

Take the derivative of $p = mv$ with respect to t to get

$$\frac{dp}{dt} = m\frac{dv}{dt}$$

but since acceleration is the derivative of velocity, this gives

$$F = ma.$$

(b) Derive a formula for the force F if the mass is not assumed to be constant.

We do the same thing as in part (a), except this time mv is a product of two functions of t. Therefore we get

$$F = \frac{dp}{dt} = v\frac{dm}{dt} + ma.$$