8. (2+10 points) Over the summer you are hired by a trucking company to help them improve operations. A truck driver is paid \$12 per hour for driving a truck over a 200 mile stretch of highway. The cost of driving the truck at an average velocity of v miles per hour is $(5 + .568v)$ dollars. The truck driver must drive between 40 mph and 70 mph.

(a) If the truck driver drives an average of v miles per hour for 200 miles, how long does he drive?

$$t = \frac{200}{v}\text{hours}$$

(b) At what average speed should the truck driver be told to drive in order to minimize the company's cost for the 200-mile trip? Note that the company's cost is the cost of paying the driver plus the cost of driving the truck.

The cost function is given by

$$C(v) = 12\left(\frac{200}{v}\right) + (5 + 0.568v).$$

Since we would like to minimize this function, we take the derivative and set it equal to 0:

$$\begin{aligned} C'(v) &= -\frac{2400}{v^2} + .568 = 0 \\ v &= \pm 65\text{mph-(but we can discard the negative.)} \end{aligned}$$

To see that this is a minimum, observe that $C''(65) > 0$. Now we just need to check this against the endpoints. $C(65) = \$78.84$, $C(40) = \$87.72$, and $C(70) = \$79.04$. So the driver should be told to drive an average of 65 miles per hour in order to minimize the cost to the company.