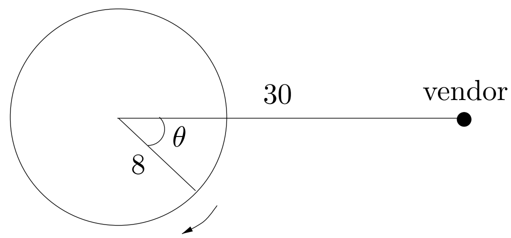
9. (2+4+6 points) You have been searching for the cotton candy vendor all day at the carnival. As you board the merry-go-round, you spot the candy man. Unfortunately, you are stuck on the merry-go-round. The vendor's stand is 30 feet from the center of the merry-go-round, and you begin your ride directly on the line of sight between the center of the merry-go-round and the vendor. The merry-go-round has a radius of 8 feet and is turning at a rate of $\frac{\pi}{60}$ radians/second.



(a) How long does it take for the merry-go-round to rotate $\frac{\pi}{6}$ radians?

$t = 10$ seconds.

(b) How far are you from the vendor when the merry-go-round has rotated $\frac{\pi}{6}$ radians? [The law of cosines may help here. It states that given a triangle of side lengths a, b, and c with angle θ between sides a and b, then one has $c^2 = a^2 + b^2 - 2ab\cos\theta$.]

Use the law of cosines with $a = 8$, $b = 30$, $\theta = \frac{\pi}{6}$, and c the distance between you and the vendor. So $c = 23.42$ feet.

(c) How fast is the distance between you and the vendor changing when the merry-go-round has rotated $\frac{\pi}{6}$ radians?

Take the derivative of the law of cosines with respect to t:

$$2c\frac{dc}{dt} = 2ab\sin(\theta)\frac{d\theta}{dt}.$$

Solving this equation for $\frac{dc}{dt}$ and using that $a = 8$, $b = 30$, $c = 23.42$, $\theta = \frac{\pi}{6}$, and $\frac{d\theta}{dt} = \frac{\pi}{60}$ we obtain that $\frac{dc}{dt} = 0.27$ feet/second.