1. (16 points) Use the information given below to answer the following questions. Show work where appropriate.

x	0	1	2	3	4
$f(x)$	0.5	2	2.5	0	-4
$f'(x)$	1.5	0.5	-1	-3	-3.5

(a) If $g(x) = Ax^2$ for some constant A, find $h'(2)$ where $h(x) = g(x) + f(2x)$. Your answer may involve the constant A.

We have $h'(x) = g'(x) + 2f'(2x)$ and therefore, $h'(2) = g'(2) + 2f'(4)$. Also, $g'(2) = 4A$, so $h'(2) = 4A - 7$.

(b) Suppose $k(x) = 4^{f(x)}$. Find $k'(1)$.

By the chain rule, $k'(x) = ln(4) \cdot f'(x) \cdot 4^{f(x)}$ and thus, $k'(1) = ln(4) \cdot f'(1) \cdot 4^{f(1)} = 16 \cdot 0.5 \cdot ln(4) = 8ln(4)$.

(c) Suppose $l(x)$ is a linear function of x, $l(4) = 0$, and $l'(4) < f'(4)$. Which of the following is true about $l(x)$? (Circle all that apply; you need not justify your answer):

(i) $l(x) > 0$ for $x > 4$.

(ii) $l(x) < 0$ for $x > 4$.

(iii) $l(x)$ is increasing for all x.

(iv) $l(x)$ is decreasing for all x.

Since $l'(4) < f'(4) = -3.5$, we see that $l(x)$ is decreasing. Since $l(x)$ is a line it is decreasing for all x. Because $l(4) = 0$, it also follows that $l(x) < 0$ for $x > 4$, so (ii) and (iv) are correct.

(d) Suppose $j(x)$ is an exponential function and that $j(0) = 1$. Let $h(x) = j(x)f(x)$. If $h'(0) = 7$, find a formula for $j(x)$.

Because $j(x)$ is an exponential with $j(0) = 1$ we know $j(x) = b^x$ for some constant b. By the product rule, $h'(x) = j'(x)f(x) + j(x)f'(x)$, so we have an equation $7 = j'(0) \cdot 0.5 + 1.5$. It follows that $j'(0) = 11$. But $j'(0) = \ln(b)b^0 = \ln(b)$. So $b = e^{11}$ and therefore $j(x) = e^{11x}$.