10. (12 points) You are in charge of ticket sales for the U-M/Ohio State football game next year. Fans can buy pre-season tickets prior to September 1, 2006 for \$22.50 each. After September 1st, the price will be \$25 per ticket. The \$25 tickets are called term tickets. It turns out that pre-season ticket sales are a good predictor of term ticket sales, though the relationship is somewhat complicated. The number of term tickets sold, $T(x)$ (in thousands), is a function of the number of pre-season tickets sold, x (in thousands), and is given by:

$$T(x) = -0.02x^2 + 1.9x + 8.$$

Assume that the maximum capacity of the stadium is 115,000. What number of pre-season and term tickets should be sold to maximize revenue? Be sure to completely justify your answers–using techniques of calculus–(i.e., merely a graph or table will not suffice).

Since x is the number of pre-season tickets sold and $T(x)$ is the number of term tickets sold, the total revenue, $R(x)$, is given by

$$R(x) = 22.5x + 25T(x).$$

We must find the global maximum of $R(x)$ on the interval $0 \leq x \leq 115$. We start by finding the critical points of R, and these occur where $R'(x) = 0$ (note that $R(x)$ is a quadratic polynomial so that it and it's derivative are defined everywhere). By direct calculation,

$$R'(x) = 22.5 + 25(-0.04x + 1.9) = 22.5 - x + 47.5$$

When we set $R'(x) = 0$ and solve for x we get $x = 70$, the single critical point of $R(x)$. Since $R(x)$ is a parabola opening *down*, $x = 70$ is in fact the global maximum of $R(x)$. Furthermore, $T(70) = 43$ (and observe that $70 + 43 = 113 < 115$ so that this falls within the stadium capacity). Therefore, to maximize revenue, U-M should sell $70,000$ pre-season tickets and $43,000$ term tickets.