6. (6 points) Consider the function \( f(x) = 3xe^{ax} + x^2 \), where \( a \) is a constant. If the error in the linear approximation to \( f(x) \) near \( x = 0 \) is 0.02 when \( x = 0.1 \), what is \( a \)? Show your work.

First notice that \( f(0) = 0 \). We compute the derivative using the product and chain rules. We get:

\[
f'(x) = 3e^{ax} + 3axe^{ax} + 2x
\]

It follows that \( f'(0) = 3 \) and so the equation of the tangent line is \( g(x) = 3x \). The error is defined as

\[
\text{Error} = f(0.1) - \text{linear approximation at } x = 0.1
\]

so plugging in \( x = 0.1 \), we get the following equation:

\[
0.02 = (0.3e^{0.1a} + 0.01) - 0.3
\]

When we solve this for \( a \) we find that \( a \sim 0.3279 \).

7. (6 points) The kinetic energy, \( K \) in Joules, of a particle in motion is a function of its fixed mass, \( M \) in kg, and its velocity, \( v \), in \( \frac{m}{s} \), and is given by:

\[
K = \frac{1}{2} M v^2.
\]

For an object with a mass of 2 kg, how fast is its kinetic energy increasing when it is traveling 3 m/s and accelerating at a rate of 10 m/s\(^2\)?

We differentiate the Kinetic energy equation with respect to time. Note that the mass, \( M \), is fixed, and therefore is a constant with respect to time.

\[
\frac{dK}{dt} = \frac{1}{2} 2Mv \frac{dv}{dt} = Mv \frac{dv}{dt}
\]

We now plug in \( M = 2 \), \( v = 3 \), \( \frac{dv}{dt} = 10 \) and solve for \( \frac{dK}{dt} \). We get \( \frac{dK}{dt} = 60 \) Joules/sec (note that a Joule is the same as a \( \frac{kg \cdot m^2}{s^2} \)).