6. (6 points) Consider the function $f(x)=3 x e^{a x}+x^{2}$, where $a$ is a constant. If the error in the linear approximation to $f(x)$ near $x=0$ is 0.02 when $x=0.1$, what is $a$ ? Show your work.

First notice that $f(0)=0$. We compute the derivative using the product and chain rules. We get:

$$
f^{\prime}(x)=3 e^{a x}+3 a x e^{a x}+2 x
$$

It follows that $f^{\prime}(0)=3$ and so the equation of the tangent line is $g(x)=3 x$. The error is defined as

$$
\text { Error }=f(0.1)-\text { linear approximation at } x=0.1
$$

so plugging in $x=0.1$, we get the following equation:

$$
0.02=\left(0.3 e^{0.1 a}+0.01\right)-0.3
$$

When we solve this for $a$ we find that $a \sim 0.3279$.
7. (6 points) The kinetic energy, $K$ in Joules, of a particle in motion is a function of its fixed mass, $M$ in kg , and its velocity, $v$, in $\frac{m}{s}$, and is given by:

$$
K=\frac{1}{2} M v^{2}
$$

For an object with a mass of 2 kg , how fast is its kinetic energy increasing when it is traveling $3 \frac{\mathrm{~m}}{\mathrm{~s}}$ and accelerating at a rate of $10 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ ?

We differentiate the Kinetic energy equation with respect to time. Note that the mass, $M$, is fixed, and therefore is a constant with respect to time.

$$
\frac{d K}{d t}=\frac{1}{2} 2 M v \frac{d v}{d t}=M v \frac{d v}{d t}
$$

We now plug in $M=2, v=3, \frac{d v}{d t}=10$ and solve for $\frac{d K}{d t}$. We get $\frac{d K}{d t}=60 \mathrm{Joules} / \mathrm{sec}$ (note that a Joule is the same as a $\frac{\mathrm{kg} \cdot \mathrm{m}^{2}}{\mathrm{~s}^{2}}$ ).

