6. (6 points) Consider the function $f(x) = 3xe^{ax} + x^2$, where a is a constant. If the error in the linear approximation to $f(x)$ near $x = 0$ is 0.02 when $x = 0.1$, what is a? Show your work.

First notice that $f(0) = 0$. We compute the derivative using the product and chain rules. We get:

$$f'(x) = 3e^{ax} + 3axe^{ax} + 2x$$

It follows that $f'(0) = 3$ and so the equation of the tangent line is $g(x) = 3x$. The error is defined as

$$Error = f(0.1) - \text{linear approximation at} x = 0.1$$

so plugging in $x = 0.1$, we get the following equation:

$$0.02 = (0.3e^{0.1a} + 0.01) - 0.3$$

When we solve this for a we find that $a \sim 0.3279$.

7. (6 points) The kinetic energy, K in Joules, of a particle in motion is a function of its fixed mass, M in kg, and its velocity, v, in $\frac{m}{s}$, and is given by:

$$K = \frac{1}{2}Mv^2.$$

For an object with a mass of 2 kg, how fast is its kinetic energy increasing when it is traveling $3\frac{m}{s}$ and accelerating at a rate of $10\frac{m}{s^2}$?

We differentiate the Kinetic energy equation with respect to time. Note that the mass, M, is fixed, and therefore is a constant with respect to time.

$$\frac{dK}{dt} = \frac{1}{2}2Mv\frac{dv}{dt} = Mv\frac{dv}{dt}$$

We now plug in $M = 2$, $v = 3$, $\frac{dv}{dt} = 10$ and solve for $\frac{dK}{dt}$. We get $\frac{dK}{dt} = 60$ Joules/sec (note that a Joule is the same as a $\frac{kg \cdot m^2}{s^2}$).