2. (20 points) Suppose f, g, and h are all differentiable functions of x , $f(x)$ and $g(x)$ are positive for all x, and that a, and b are positive constants. Your answers below will be in terms of f, g, h (and/or their derivatives) and perhaps the constants a or b.

(a) Find $\frac{dy}{dx}$ if $y = f(2) + \ln(f(x^2))$.

$$\frac{dy}{dx} = \frac{f'(x^2)2x}{f(x^2)}$$

(b) Find $\frac{dy}{dx}$ if $y = f(x^a + 2x) + 2^{g(x)}$.

$$\frac{dy}{dx} = f'(x^a + 2x)(ax^{a-1} + 2) + \ln 2(2^{g(x)})g'(x)$$

(c) Find $\frac{dy}{dx}$ if $y = \frac{h(bx)}{\cos(x) + 2}$.

$$\frac{dy}{dx} = \frac{bh'(bx)(\cos(x) + 2) + h(bx)\sin(x)}{(\cos(x) + 2)^2}$$

(d) If $f'(x) = ag(x)$ and $g'(x) = -af(x)$, when is $y = f(x)g(x)$ increasing? [Refer to the instructions above for conditions on f, g and a.] Justify your answer.

$$\begin{aligned}\frac{dy}{dx} &= f'(x)g(x) + f(x)g'(x), \\ &= a(g(s))^2 - a(f(x))^2 = a\left((g^2(x) - f^2(x)\right), \\ &= a\left(g(x) + f(x)\right)\left(g(x) - f(x)\right).\end{aligned}$$

Thus, since a, f and g are all positive, $\frac{dy}{dx} > 0$ when $g(x) > f(x)$.