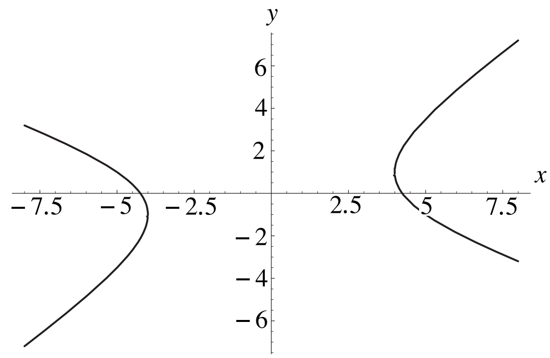
3. (18 points) Below is a graph of the curve implicitly defined by the equation

$$2y^2 - xy - x^2 = -18.$$



(a) Find a formula for $\frac{dy}{dx}$ as a function of both x and y.

Using implicit differentiation, we have

$$4y\frac{dy}{dx} - y - x\frac{dy}{dx} - 2x = 0, \text{ so } (4y - x)\frac{dy}{dx} = y + 2x$$

which gives $\boxed{\frac{dy}{dx} = \frac{y+2x}{4y-x}.}$

(b) Find the value of $\frac{dy}{dx}$ at the point $(5, -1)$.

$$\frac{dy}{dx}\Big|_{(5,-1)} = \frac{-1+10}{-4-5} = -\frac{9}{9} = -1$$

(c) Find any points (x_0, y_0) where $\frac{dy}{dx}$ is undefined, or give justification why no such points exist.

From above, we know $\frac{dy}{dx}$ is undefined if $4y = x$.

Thus,

$$2y^2 - 4y^2 - 16y^2 = -18,$$

$$\text{so} \quad -18y^2 = -18; \text{ or } y^2 = 1 \text{ which gives } y = \pm 1.$$

If $y = \pm 1$, and $4y = x$, then $x = \pm 4$. $\boxed{\text{The points are } (4, 1) \text{ and } (-4, -1).}$

(d) Find any points (x_0, y_0) where $\frac{dy}{dx} = 0$, or give justification why no such points exist.

The expression for $\frac{dy}{dx}$ will be zero if $y = -2x$, so

$$2(4x^2) + 2x^2 - x^2 = 9x^2 = -18.$$

However, this gives $x^2 = -2$, and there are no real solutions. Thus, the graph has no horizontal tangents, or there are no real values such that $\frac{dy}{dx} = 0$.