3. (18 points) Below is a graph of the curve implicitly defined by the equation

\[ 2y^2 - xy - x^2 = -18. \]

(a) Find a formula for \( \frac{dy}{dx} \) as a function of both \( x \) and \( y \).

Using implicit differentiation, we have

\[ 4y \frac{dy}{dx} - y - x \frac{dy}{dx} - 2x = 0, \]

so \( (4y - x) \frac{dy}{dx} = y + 2x \)

which gives \( \frac{dy}{dx} = \frac{y + 2x}{4y - x} \).

(b) Find the value of \( \frac{dy}{dx} \) at the point \((5, -1)\).

\[ \frac{dy}{dx} \bigg|_{(5,-1)} = \frac{-1 + 10}{-4 - 5} = \frac{9}{9} = -1 \]

(c) Find any points \((x_0, y_0)\) where \( \frac{dy}{dx} \) is undefined, or give justification why no such points exist.

From above, we know \( \frac{dy}{dx} \) is undefined if \( 4y = x \).

Thus,

\[ 2y^2 - 4y^2 - 16y^2 = -18, \]

so \( -18y^2 = -18; \) or \( y^2 = 1 \) which gives \( y = \pm 1 \).

If \( y = \pm 1 \), and \( 4y = x \), then \( x = \pm 4 \). The points are \((4, 1)\) and \((-4, -1)\).

(d) Find any points \((x_0, y_0)\) where \( \frac{dy}{dx} = 0 \), or give justification why no such points exist.

The expression for \( \frac{dy}{dx} \) will be zero if \( y = -2x \), so

\[ 2(4x^2) + 2x^2 - x^2 = 9x^2 = -18. \]

However, this gives \( x^2 = -2 \), and there are no real solutions. Thus, the graph has no horizontal tangents, or there are no real values such that \( \frac{dy}{dx} = 0 \).