

4. (12 points) Ellen and Renzo ran the Detroit marathon last weekend. The distance Ellen traveled (in meters) is given by  $E(t)$  where  $t$  is time measured in seconds since the start of the race. Similarly, the distance in meters Renzo traveled is given by the function  $R(t)$ . For  $x$  measured in meters let  $F(x) = R(E^{-1}(x))$ . Assume that Ellen moves forward throughout the race—she does not even take a rest!

- (a) What is the practical interpretation of  $F(50)$ .

We have

$$F(50) = R(E^{-1}(50))$$

which gives the distance, in meters, that Renzo has traveled when Ellen has traveled 50 meters.

- (b) After the initial blast of speed from her start, Ellen ran at a constant rate of 5 meters per second for  $2 < t < 10$ , and she had run a distance of 39 meters after 7 seconds. Renzo wore a device that tracked the distance he had run at one second intervals. The data he collected is summarized in the table below.

$t$	0	1	2	3	4	5	6	7	8	9	10
$R(t)$	0	10	16	22	28	34	40	46	52	58	64

Use any of the information above to approximate  $F'(39)$ .

$$F'(39) = R'(E^{-1}(39))(E^{-1})'(39) = R'(7)(E^{-1})'(39).$$

From the table, we approximate  $R'(7) = 6$ , and we know  $E^{-1}(39) = 7$  and  $E'(7) = 5$ . Thus, since

$$(E^{-1})'(39) = \frac{1}{E'(E^{-1}(39))} = \frac{1}{E'(7)} = \frac{1}{5},$$

we have  $F'(39) = \frac{6 \text{ meters (for Renzo)}}{5 \text{ meters (for Ellen)}}$ .

- (c) Give a practical interpretation of  $F'(39)$ .

Since  $F'(39)$  is in  $\frac{\text{meters for Renzo}}{\text{meters for Ellen}}$ , once Ellen has run 39 meters, if Ellen's distance increases by 1 meter, Renzo will travel approximately 1.2 meters.