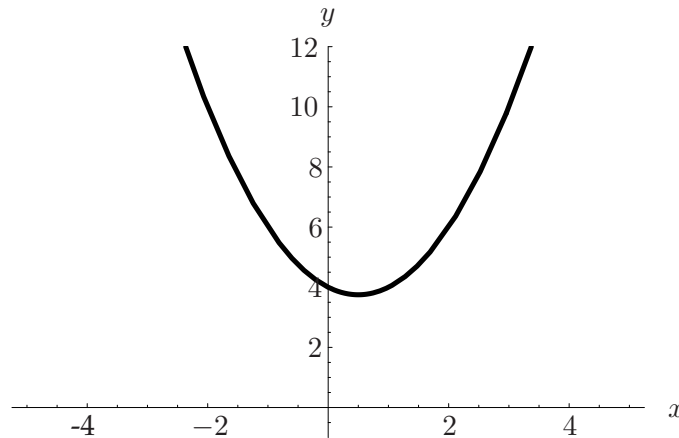


1. (10 points) Using the graph in the figure below to help you, find the equations of all lines through the origin and tangent to the parabola

$$y = x^2 - x + 4.$$

[Show all relevant work—graphical approximations are not sufficient. Please circle your answers.]



The equation of any line, l , through the origin is $l = mx$ since the y -intercept is zero. A line that is tangent to the curve must have the same slope as the original curve at a given point, so

$$y' = 2x - 1 = m$$

Therefore, at a point $x = a$ the equations of the lines tangent to the curve must satisfy

$$\underbrace{a^2 - a + 4}_y = \underbrace{(2a - 1)}_m \cdot a$$

which implies $a^2 = 4$, so $a = \pm 2$.

If $a = x = 2$, $y = 6$, and $m = 3$, so the equation of the line is $y = 3x$.

If $a = x = -2$, $y = 10$, and $m = -5$, so the equation of the line is $y = -5x$.