1. (10 points) Using the graph in the figure below to help you, find the equations of all lines through the origin and tangent to the parabola

\[ y = x^2 - x + 4. \]

[Show all relevant work—graphical approximations are not sufficient. Please circle your answers.]

The equation of any line, \( l \), through the origin is \( l = mx \) since the \( y \)-intercept is zero. A line that is tangent to the curve must have the same slope as the original curve at a given point, so

\[ y' = 2x - 1 = m. \]

Therefore, at a point \( x = a \) the equations of the lines tangent to the curve must satisfy

\[ a^2 - a + 4 = (2a - 1) \cdot a \]

\[ y = m \cdot x \]

which implies \( a^2 = 4 \), so \( a = \pm 2 \).

If \( a = x = 2, y = 6, \) and \( m = 3 \), so the equation of the line is \( y = 3x \).

If \( a = x = -2, y = 10, \) and \( m = -5 \), so the equation of the line is \( y = -5x \).