2. (20 points) Suppose \( f \) and \( g \) are differentiable functions with the following values:

\[
    f(0) = 3, \quad f'(0) = 4, \quad g(0) = -1, \quad \text{and} \quad g'(0) = 2.
\]

Show your work on the following:

(a) Find \( h'(0) \) given \( h(x) = \frac{g(x)}{f(x)} \).

\[
h'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f^2(x)} \quad \text{thus} \quad h'(0) = \frac{(2)(3) - (-1)(4)}{9} = \frac{10}{9}
\]

(b) i. Find \( k'(0) \) given \( k(x) = (g(x))^2 f(x) \).

\[
k'(x) = 2g(x)g'(x)f(x) + (g(x))^2 f'(x).
\]

Thus \( k'(0) = 2(-1)(2)(3) + (-1)^2 4 = -8 \)

ii. Determine the local linearization of \( k(x) \) near \( x = 0 \), and use that to approximate \( k(0.001) \).

Near \( x = 0 \), \( k(x) \approx k(0) + k'(0)(x - 0) \),

and \( k(0) = (-1)^2(3) = 3 \).

Therefore \( k(0.001) \approx 3 + (-8)(0.001) = 2.992 \).

(c) Find \( m'(0) \) given \( m(x) = \sin \left( (f(x))^3 \right) \).

\[
m'(x) = \cos \left( (f(x))^3 \right) \left( 3 \ (f(x))^2 \ f'(x) \right),
\]

so \( m'(0) = \cos(27) \cdot (27)(4) = 108 \cos(27) \).