2. (20 points) Suppose $f$ and $g$ are differentiable functions with the following values:

$$f(0) = 3, \quad f'(0) = 4, \quad g(0) = -1, \quad \text{and} \quad g'(0) = 2.$$

Show your work on the following:

(a) Find $h'(0)$ given $h(x) = \dfrac{g(x)}{f(x)}$.

$h'(x) = \dfrac{g'(x)f(x) - g(x)f'(x)}{f^2(x)}$ thus $h'(0) = \dfrac{(2)(3) - (-1)(4)}{9} = \dfrac{10}{9}$

(b) i. Find $k'(0)$ given $k(x) = (g(x))^2 f(x)$.

$k'(x) = 2g(x)g'(x)f(x) + (g(x))^2 f'(x)$.

Thus $k'(0) = 2(-1)(2)(3) + (-1)^2 4 = -8$

ii. Determine the local linearization of $k(x)$ near $x = 0$, and use that to approximate $k(0.001)$.

Near $x = 0$,

$$k(x) \approx k(0) + k'(0)(x - 0),$$

and $k(0) = (-1)^2(3) = 3$.

Therefore $k(0.001) \approx 3 + (-8)(0.001) = 2.992$.

(c) Find $m'(0)$ given $m(x) = \sin\left((f(x))^3\right)$.

$m'(x) = \cos\left((f(x))^3\right)(3\ (f(x))^2 f'(x))$,

so $m'(0) = \cos(27) \cdot (27)(4) = 108\cos(27)$.