3. (14 points) Find the values of the constants $a,\ b$ and c such that the function

$$f(x) = ax^2 + bx + c$$

"fits" the function

$$g(x) = -2\ln(x+1) + 0.5e^x + 1.5\sin(x)$$

near $x = 0$ in the sense that:

$$g(0) = f(0), \quad g'(0) = f'(0) \quad \text{and} \quad g''(0) = f''(0).$$

Show all work.

We have $g(0) = 0 + 0.5 + 0 = 1/2$ and $f(0) = c$,
so $\boxed{c = 0.5}$.

We find the first derivatives:

$$g'(x) = \frac{-2}{(x+1)} + 0.5e^x + 1.5\cos(x), \quad \text{and} \quad f'(x) = 2ax + b.$$

Thus, $g'(0) = -2 + 0.5 + 1.5 = 0, \quad$ and $\quad f'(x) = b$,

so $\boxed{b = 0}$.

Second derivatives give:

$$g''(x) = \frac{2}{(x+1)^2} + 0.5e^x - 1.5\sin(x) \quad \text{and} \quad f''(x) = 2a.$$

Thus,
$g''(0) = 2 + 0.5 - 0 = 2.5 \quad$ and $\quad f''(0) = 2a$,

so $2a = 2.5$ which gives $\boxed{a = 1.25.}$

$a =$ 1.25

$b = 0$ 0

$c =$ 0.5