6. (5 points) Parasitoids are insects that lay eggs in, on, or close to other (host) insects. Their larvae then devour the host insect, resulting in the death of the host. The likelihood of escaping parasitism may depend on parasitoid density. One such model sets the probability, P, of escaping parasitism as:

$$P = f(D) = \left(1 + \frac{a\,D}{k}\right)^{-k}$$

where D is the parasitoid density and a and k are positive constants.

Determine whether the probability of escaping parasitism increases or decreases as parasitoid density increases. Justify your answer.

First we consider the derivative of P with respect to D:

$$P' = -a\left(1 + \frac{aD}{k}\right)^{-k-1}$$

which is negative, thus P decreases as D increases.

7. (12 points) Let $P = f(t)$ be the total amount, in trillions of barrels, of the world's reserves of petroleum in year t.

   (a) What does the statement $f(2006) = 1.2$ tell you about the petroleum reserves?

   In 2006 there were 1.2 trillion barrels in the world's petroleum reserves.

   (b) Evaluate and interpret $f^{-1}(1.2)$.

   This gives the year in which there was 1.2 trillion barrels of petroleum in the world's reserve and from above we have that was the year 2006.

   (c) What does the statement $f'(2006) = -0.003675$ tell you about the petroleum reserves?

   This tells us that in 2006 the world's reserves of petroleum was decreasing at the rate of 0.003675 trillion barrels per year. This means that in 2007 the world would have approximately 1.2-0.003675 = 1.196 trillion barrels of petroleum in its reserves.

   (d) Evaluate and interpret $\left(f^{-1}\right)'(1.2)$.

   This value can be found as

$$\left(f^{-1}\right)'(t) = \frac{1}{(f'(f^{-1}(t)))}.$$

   Therefore $\left(f^{-1}\right)'(1.2)^{-1} = \left(f'(f^{-1}(1.2)\right) = -(0.003675)^{-1} = -272.11$ years per trillion barrels. This states that at the 2006 rate it would take approximately 272 years to use up a trillion barrels of the world's petroleum reserves. [Note that the rate of depletion is expected to increase greatly in the next few years—*i.e.*, f' becomes more negative—so this expectation is not reasonable.]