8. (17 points)

(a) A small company makes x hand-painted tiles daily at a cost of $C(x) = 125 + 30x + 2x^{3/2}$ dollars. What daily production level minimizes the *average* cost—*i.e.*, the cost per tile?

First define the average cost

$$A(x) = \frac{C(x)}{x} = \frac{125}{x} + 30 + 2x^{1/2}.$$

We find $A'(x)$ and any critical points:

$$A'(x) = -\frac{125}{x^2} + x^{-1/2}.$$

Thus A' is undefined if $x = 0$ (not in the domain of A), and $A'(x) = 0$ if $x = 25$.

Thus, we have one critical point at $x = 25$.

Note that

$$A''(x) = \frac{250}{x^3} - 0.5x^{-3/2},$$

and $A''(25) > 0$. Thus, $x = 25$ is a local min, and since this is the only critical point and the function is continuous on its domain, $x = 25$ is the absolute minimum.

(b) If the the company sells each tile for \$75, how many tiles should they make daily in order to maximize daily profit?

First define the profit function $\pi(x) = R(x) - C(x)$, ie

$$\pi(x) = 75x - 125 - 30x - 2x^{3/2} = -125 + 45x - 2x^{3/2}.$$

The critical points occur when $\pi'(x) = 0$ or is undefined. For x in the domain, the only critical point is when $\pi(x) = 0$, or

$$\pi'(x) = 45 - 3x^{1/2} = 0, \text{ giving } x = 225.$$

Once again we must test the critical point. The second derivative here is

$$\pi''(x) = -\frac{3}{2}x^{-1/2}$$

which is negative for $x > 0$, so the critical point gives a local max. Since it is the only critical point on the domain and the function is continuous, the maximum profit occurs when the company makes 225 tiles per day.