2. In 1956, Marion Hubbert began a series of papers predicting that the United States' oil production would peak and then decline. Although he was criticized at the time, Hubbert's prediction was remarkably accurate. He modeled the annual oil production $P(t)$, in billions of barrels of oil, over time t, in years, as the *derivative* of the *logistic function* $Q(t)$ given below—*i.e.*, $Q'(t) = P(t)$. The function P is measured in years since the middle of 1910.

   The function $Q(t)$ is given by

$$Q(t) = \frac{Q_0}{1 + ae^{-bt}}\ , \text{where } a, b, Q_0 > 0. \tag{1}$$

   For your convenience, the first and second derivatives of $Q(t)$ are given as well:

$$Q'(t) = -\frac{Q_0}{(1 + ae^{-bt})^2}\left(-abe^{-bt}\right) = \frac{abQ_0e^{-bt}}{(1 + ae^{-bt})^2},$$

   and

$$Q''(t) = \frac{ab^2Q_0e^{-bt}}{(1 + ae^{-bt})^3}\left[ae^{-bt} - 1\right].$$

   (a) (2 points) Interpret, in the context of this problem, $P'(56)$.

   (b) (6 points) Determine the year of maximum annual production t_{max}. Your answer may involve all or some of the constants a, b, Q_0.

   (c) (2 points) Find the maximum annual production $P(t_{max})$. Again, your answer may involve all or some of the constants a, b, Q_0.

   (d) (2 points) In his 1962 paper, Hubbert studied the available data on oil production to date and concluded that $a = 46.8$, $b = 0.0687$, and $Q_0 = 170$ Bb (billion barrels). Using your results from part (b), when would Hubbert's curve predict the peak in US oil production? (The actual peak occurred in 1964.)