3. Use the information below to find an equation that best models the situation and most accurately fits the given data.

   (a) i. (2 points) Suppose a pair of shoes at DSW costs \$50 after a 10% discount. Find a formula for $P(n)$, the price of the shoes after n discounts of 10%, where $n \geq 0$.

   Since each successive discount makes the price 90% of the previous price, the function $P(n)$ is exponential. Moreover, we are given that $P(1) = 50$. Thus, using $P(n) = P_0 a^n$, we know that $a = 0.90$, and that $50 = P_0(0.90)$. From here we get $P_0 = (50/0.90)$, and arrive at $P(n) = (\frac{50}{0.90})(0.90)^n$.

   ii. (4 points) Find and interpret $P'(4)$ in the context of this problem.

   $P'(n) = \ln(0.90)(\frac{50}{0.90})(0.90)^n$, from which we find that $P'(4) \approx -3.84$ \$/discount. The units give us a hint for the interpretation: After the 4th discount, the 5th discount will lower the price of the shoes by approximately an additional \$3.84.

   (b) (6 points) Michigan's population (in millions) for the last three years as measured by the U.S. Census Bureau is given below.

Year	2005	2006	2007
Population	10.108	10.102	10.071

   Find a formula to approximate the population of Michigan, $P(t)$, with t in years since 2005. Using this information, approximate the population of Michigan in 2008. Show your work.

   We may approximate $P(t)$ by a linear function or an exponential function—neither exactly fits the data. Multiple answers were accepted here. [Note: the grading on this problem was based on the reasonableness of your answer to the first part of the question (and reasoning)– and, if that reasoning is sound, the second part of the answer was based on the first answer– units must be included on the 2nd part of the answer.]

   (c) (6 points) The height $h(t)$ (in ft. above the ground) of a passenger on a ferris wheel (a circular fair ride) varies from a maximum of 50 ft. to a minimum of 2 ft. as a function of time t (in minutes). If the ferris wheel makes 0.1 revolutions/minute, and the passenger is initially at the top of the ride, find a formula for the vertical velocity of the passenger, $v(t)$.

   The vertical velocity $v(t)$ is the derivative of the vertical position $h(t)$, so we must find that first. To find $h(t)$, we can find the midline by taking the average of the max and min, which in this case ends up being $(50 + 2)/2 = 26$. One way to find the amplitude is to subtract the midline from the maximum, so $50 - 26 = 24$. Also, since t is measured from the top of the ride (the maximum value of $h(t)$), then $h(t)$ is most easily modeled by a cosine function. Lastly, since the ride completes 0.1 revolutions/minute, then it completes 1 revolution in 10 minutes, making the period 10 minutes, from which we get the coefficient of t to be $2\pi/10 = \pi/5$. Putting all this together, we have $h(t) = 24\cos(\frac{\pi}{5}t) + 26$. with t in minutes. Thus, $v(t) = -\frac{24\pi}{5}\sin(\frac{\pi}{5}t)$.