4. (8 points) Determine a and b for the function of the form $y = f(t) = at^2 + b/t$, with a local minimum at (1,12).

Differentiating $y = at^2 + b/t$, we have

$$\frac{dy}{dt} = 2at - bt^{-2}.$$

Since there must be a critical point when $t = 1$, we have $2a - b = 0$, so $b = 2a$. Substituting and using the point (1,12) in the original equation, we have $12 = a + b = a + 2a = 3a$. Thus $a = 4$, and $b = 2(4) = 8$, and $y = 4t^2 + 8/t$.

To show that the point (1,12) is a local minimum, we use the second derivative:

$$\frac{d^2y}{dt^2} = 8 + \frac{16}{t^3},$$

which is positive for $t = 1$. Thus, the critical point (1,12) is indeed a local minimum.

5. (6 points) The circulation time of a mammal (that is, the average time it takes for all the blood in the body to circulate once and return to the heart) is proportional to the fourth root of the body mass of the mammal. The constant of proportionality is 17.40 if circulation time is in seconds and body mass is in kilograms. The body mass of a certain growing child is 45 kg and is increasing at a rate of 0.1 kg/month. What is the rate of change of the circulation time of the child?

If we let C represent circulation time in seconds and B represent body mass in kilograms, we have $C = 17.40B^{1/4}$. As the child grows, both the body mass and the circulation time change over time. Differentiating with respect to time t, in months, we have $\frac{dC}{dt} = 17.40\left(\frac{1}{4}B^{-3/4}\frac{dB}{dt}\right)$. Substituting $B = 45$ and $dB/dt = 0.1$, we have

$$\begin{aligned} \frac{dC}{dt} &= 17.40\left(\frac{1}{4}(45)^{-3/4}(0.1)\right) \\ &= 0.025. \end{aligned}$$

The circulation time is increasing at a rate of 0.025 seconds per month.