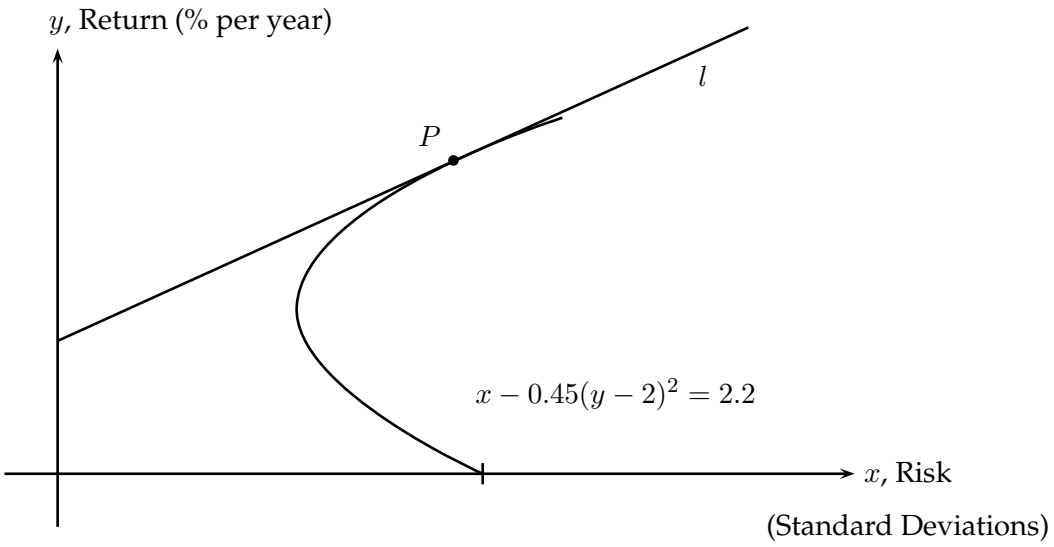
6. In Modern Portfolio Theory, a client's portfolio is structured in a way that balances risk and return. For a certain type of portfolio, the risk, x, and return, y, are related by the equation $x - 0.45(y-2)^2 = 2.2$. This curve is shown in the graph below. The point P represents a particular portfolio of this type with a risk of 3.8 units. The tangent line, l, through point P is also shown.



(a) (5 points) Using implicit differentiation, find dy/dx, and the coordinates of the point(s) where the slope is undefined.

Taking the derivative with respect to x of the equation yields $1 - 0.9(y-2)\frac{dy}{dx} = 0$. Solving this yields

$$\frac{dy}{dx} = \frac{10}{9(y-2)},$$

which is undefined when $y = 2$. Plugging $y = 2$ into the equation of the curve, we obtain $x = 2.2$ giving $(2.2, 2)$ as the point on the curve where the slope is undefined.

(b) (8 points) The y-intercept of the tangent line for a given portfolio is called the *Risk Free Rate of Return*. Use your answer from (a) to find the Risk Free Rate of Return for this portfolio.

We use the equation of the curve to find the y-value at P to be about 3.8856. Using this and the information from part (a) above, we get the slope of the tangent line to be about 0.5892. Thus, the equation of the tangent line can be found by using the point-slope formula, $y - 3.8856 = 0.5892(x - 3.8)$. Now, since the Risk Free Rate of Return is the y-intercept, we simply set $x = 0$ to get $y \approx 1.6464$.

(c) (3 points) Now, estimate the return of an optimal portfolio having a risk of 4 units by using your information from part (b). Would this be an overestimate or an underestimate? Why?

We can use the equation of the tangent line to approximate the return of the optimal portfolio, $y \approx 3.8856 + 0.5892(4 - 3.8) = 4.0034$. Since the graph is concave down near P, then this would be an overestimate.