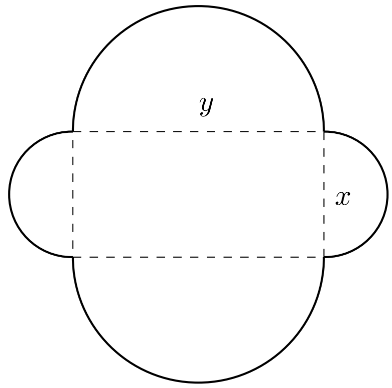
7. The figure below is made of a rectangle and semi-circles.



(a) (3 points) Find a formula for the enclosed area of the figure.

The area of the figure is the rectangular area plus the two circular areas (with radius $y/2$ and $x/2$). Thus,

$$A = xy + \pi \left(\frac{x}{2}\right)^2 + \pi \left(\frac{y}{2}\right)^2.$$

(b) (2 points) Find a formula for the perimeter of the figure.

The perimeter is the sum of the four semi-circle arcs which can be joined to make two circles of diameters x and y. So the perimeter is the sum of their circumferences , or $\pi(x + y)$.

(c) (8 points) Find the values of x and y which will maximize the area if the perimeter is 100 meters.

Since this is a constrained maximization problem, we can use the constraint, $100 = \pi(x + y)$ to eliminate one variable in the area equation. Thus, we can substitute $y = \frac{100}{\pi} - x$ into A to obtain

$$A(x) = x\left(\frac{100}{\pi} - x\right) + \pi\left(\frac{x}{2}\right)^2 + \pi\left(\frac{\frac{100}{\pi} - x}{2}\right)^2.$$

Note that we know that $0 \le x, y \le \frac{100}{\pi}$.

The above equation simplifies to

$$A(x) = \left(\frac{\pi}{2} - 1\right)x^2 + \left(\frac{100}{\pi} - 50\right)x + \frac{2500}{\pi}.$$

By solving $A'(x) = 0$, we obtain the only critical point at $x = \frac{50-\frac{100}{\pi}}{\pi-2}$. Along with the end-points $x = 0$ and $x = \frac{100}{\pi}$, we can evaluate the area $A(x)$ at each of these points and conclude that $A(\frac{100}{\pi})$ is the largest value. Thus, $(x, y) = (\frac{100}{\pi}, 0)$ is the location of the maximum area. [NOTE: the solution $(x, y) = (0, \frac{100}{\pi})$ works as well.]

(d) (3 points) If the cost, in dollars, of the materials to build the enclosure is given by $C(x)$ where x is in meters, and the Marginal Cost at $x = 100$ is 25, what does this mean in the context of the problem?

In the context of the problem, this means that the cost of the materials to build the enclosure when the perimeter is 101meters is approximately \$25 more than the cost for a 100 meter perimeter.