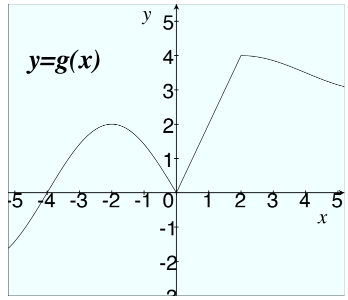
3. [14 points] Use the following table and graph to answer the questions below. Note that the graph of g passes through the points $(-2,2),(0,0)$, and $(2,4)$. All answers should be exact.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	0	1	-1	2	-1	-3	2	4	1
$f'(x)$	-1	1	-2	3	-2	2	0	3	2



a. [4 points] Let $k(x) = g(x)\arctan(f(x))$. Compute $k'(-2)$ or explain why it does not exist.

Solution: Using the Product and Chain Rules, we have

$$\begin{aligned} k'(x) &= g(x)\left(\frac{1}{1+(f(x))^2}\right)f'(x) + g'(x)\arctan(f(x)). \\ \text{So } k'(-2) &= g(-2)\left(\frac{1}{1+(f(-2))^2}\right)f'(-2) + g'(-2)\arctan(f(-2)) \\ &= (2)\left(\frac{1}{1+(-1)^2}\right)(-2) + 0(\arctan(-1)) = -2. \end{aligned}$$

Hence $k'(-2) = -2$.

b. [4 points] Let $a(x) = \dfrac{(f(x))^3}{3g(x)}$. Compute $a'(1)$ or explain why it does not exist.

Solution: Applying the Quotient and Chain Rules, we have

$$\begin{aligned} a'(x) &= \frac{3g(x)[3(f(x))^2]f'(x) - (f(x))^3(3g'(x))}{(3g(x))^2}. \\ \text{So } a'(1) &= \frac{3g(1)[3(f(1))^2]f'(1) - (f(1))^3(3g'(1))}{(3g(1))^2} \\ &= \frac{3(2)[3(-3)^2](2) - (-3)^3(3)(2)}{(3(2))^2} = \frac{27}{2} \end{aligned}$$

Hence $a'(1) = \dfrac{27}{2}$.

c. [6 points] Let $h(x) = g(g(x))$.
Find all local maxima and minima of the function h on the interval $(-4, 4)$.
Then find the global maximum and global minimum values of h on the interval $[-4, 4]$.

Solution: First, we find the critical points of h. By the Chain Rule, $h'(x) = g'(g(x))g'(x)$ so the critical points of h are values of x for which either x or $g(x)$ is a critical point of g, i.e. either $g'(x)$ or $g'(g(x))$ is zero or does not exist . Critical points of g are $x = -2$, $x = 0$, and $x = 2$. So, the critical points of h in $(-4, 4)$ are $x = -2$, $x = 0$, $x = 2$, and $x = 1$ (since $g(1) = 2$).
We check that h' is positive on the intervals $(-4, -2)$, $(0, 1)$, and $(2, 4)$ while h' is negative on the intervals $(-2, 0)$ and $(1, 2)$. By the First Derivative Test, we conclude that h has a local minimum at $x = 0$ and $x = 2$ and h has a local maximum at $x = -2$ and $x = 1$.
Since h is continuous, by the Extreme Value Theorem, h attains a global maximum and a global minimum on $[-4, 4]$. These values occur at a critical point or endpoint. Evaluating h at these points we have:

$$\begin{aligned} h(-4) &= g(g(-4)) = g(0) = 0 \\ h(-2) &= g(g(-2)) = g(2) = 4 \\ h(0) &= g(g(0)) = g(0) = 0 \\ h(1) &= g(g(1)) = g(2) = 4 \\ h(2) &= g(g(2)) = g(4) \approx 3.5 \\ h(4) &= g(g(4)) \approx g(3.5) \approx 3.7 \end{aligned}$$

Thus, the global maximum occurs at $x = -2$ and $x = 1$ with a global maximum value of 4; global minimums are at $x = -4$ and $x = 0$, and the global minimum value is 0.