6. [14 points] The force F due to gravity on a body at height h above the surface of the earth is given by

$$F(h) = \frac{mgR^2}{(R+h)^2}$$

where m is the mass of the body, g is the acceleration due to gravity at sea level ($g < 0$), and R is the radius of the earth.

a. [3 points] Compute $F'(h)$.

Solution:

$$F'(h) = \frac{-2mgR^2}{(R+h)^3}$$

b. [3 points] Compute $F''(h)$.

Solution:

$$F''(h) = \frac{6mgR^2}{(R+h)^4}$$

c. [5 points] Find the best linear approximation to F at $h = 0$.

Solution: Since $F(0) = mg$ and $F'(0) = -2mgR$, the best linear approximation to F at $h = 0$ is given by the equation

$$L(h) = mg - \frac{2mg}{R} \cdot h.$$

Hence, near $h = 0$, the linear approximation gives

$$F(h) \approx mg - \frac{2mg}{R} \cdot h.$$

d. [3 points] Does your approximation from part (c) give an overestimate or an underestimate of F? Why?

Solution: Since F'' is negative for all h (due to $g < 0$), the function is concave down, so the tangent lies above the curve and the estimate is an overestimate.