6. [14 points] The force \( F \) due to gravity on a body at height \( h \) above the surface of the earth is given by

\[
F(h) = \frac{mgR^2}{(R + h)^2}
\]

where \( m \) is the mass of the body, \( g \) is the acceleration due to gravity at sea level (\( g < 0 \)), and \( R \) is the radius of the earth.

a. [3 points] Compute \( F'(h) \).

Solution:

\[
F'(h) = \frac{-2mgR^2}{(R + h)^3}
\]

b. [3 points] Compute \( F''(h) \).

Solution:

\[
F''(h) = \frac{6mgR^2}{(R + h)^4}
\]

c. [5 points] Find the best linear approximation to \( F \) at \( h = 0 \).

Solution: Since \( F(0) = mg \) and \( F'(0) = -2mgR \), the best linear approximation to \( F \) at \( h = 0 \) is given by the equation

\[
L(h) = mg - \frac{2mg}{R} \cdot h.
\]

Hence, near \( h = 0 \), the linear approximation gives

\[
F(h) \approx mg - \frac{2mg}{R} \cdot h.
\]

d. [3 points] Does your approximation from part (c) give an overestimate or an underestimate of \( F \)? Why?

Solution: Since \( F'' \) is negative for all \( h \) (due to \( g < 0 \)), the function is concave down, so the tangent lies above the curve and the estimate is an overestimate.