**7.** [14 points] The *Kampyle of Eudoxus* is a family of curves that was studied by the Greek mathematician and astronomer Eudoxus of Cnidus in relation to the classical problem of doubling the cube. This family of curves is given by

$$a^2x^4 = b^4(x^2 + y^2).$$

where $a$ and $b$ are nonzero constants and $(x, y) \neq (0, 0)$—*i.e.*. the origin is not included.

**a.** [5 points] Find $\dfrac{dy}{dx}$ for the curve $a^2x^4 = b^4(x^2 + y^2)$.

*Solution:* Using implicit differentiation, we have

$$4a^2x^3 = 2b^4x + 2b^4y\frac{dy}{dx} \quad \text{so} \quad \frac{dy}{dx} = \frac{4a^2x^3 - 2b^4x}{2b^4y}.$$

**b.** [5 points] Find the coordinates of all points on the curve $a^2x^4 = b^4(x^2 + y^2)$ at which the tangent line is vertical, or show that there are no such points.

*Solution:* If the tangent is vertical, the slope is undefined. Setting the denominator from part (a) equal to zero gives $y = 0$. Substituting $y = 0$ in the original equation gives

$$a^2x^4 = b^4x^2$$

and since $(0, 0)$ is excluded, we know $x \neq 0$ so $x^2 = \dfrac{b^4}{a^2}$, and $x = \pm\dfrac{b^2}{a}$.
Thus, there are two points on the curve where the tangent is vertical: $\left(\dfrac{b^2}{a}, 0\right), \left(-\dfrac{b^2}{a}, 0\right)$.

**c.** [4 points] Show that when $a = 1$ and $b = 2$ there are no points on the curve at which the tangent line is horizontal.

*Solution:* Using $a = 1$ and $b = 2$ in $\dfrac{dy}{dx}$ from part (a), we have

$$\frac{dy}{dx} = \frac{4x^3 - 32x}{32y}.$$

If the tangent is horizontal, the slope is zero, so solving $4x^3 - 32x = 4x(x^2 - 8) = 0$ gives $x = 0$ or $x = \pm\sqrt{8}$. We must see if any of these values of $x$ give a point on the curve. Note that when $x = 0$, $y = 0$—and this point has been excluded from the family. If $x = \pm\sqrt{8}$, the equation of the curve gives us $64 = 16(8) + 16y^2$, which gives $y^2 = -4$, so there is no such point on the curve. Thus, there are no horizontal tangents to the curve $x^4 = 16(x^2 + y^2)$.