8. [16 points] Some airlines have started offering wireless internet access during flight. The company *WiFi Up High* (WFUH) would like to enter the market and begin working with airlines to offer such service. WFUH will charge passengers based on the amount of time the passenger uses the service during flight.

Preliminary research indicates that during NW flight 2337 (which flies from Detroit to Los Angeles), the number of hours of wifi that will be used by passengers at a price of p dollars per hour is given by the function

$$h(p) = \frac{45000}{149 + e^{0.4p}}.$$

Since offering wifi to its customers is likely to increase business for the airline, WFUH also plans to charge the airline a flat fee of a dollars per flight on which the service is offered.

a. [3 points] If WFUH offers its service to passengers at a price of \$2 per hour, what will be its expected revenue from one NW 2337 flight?

Solution: Since WFUH receives \$$a$ from the airline and expects to sell $h(2)$ hours at a price of \$2 per hour, its expected revenue will be $a + 2h(2) = a + 2\frac{45000}{149 + e^{0.8}}$, which is approximately $a + 595.14$ dollars.

b. [8 points] Use calculus to determine how much WFUH should charge passengers per hour of usage in order to maximize its revenue from NW flight 2337. (Round your answer to the nearest \$0.01.)

Solution: If WFUH charges p dollars per hour, its expected revenue, in dollars, from flight 2337 is $M(p) = a + ph(p) = a + p\dfrac{45000}{149 + e^{0.4p}}$. So, we should maximize $M(p)$ on its domain, which is $[0, \infty)$. To maximize $M(p)$, we first determine its critical points.

$$M'(p) = p\frac{-45000(0.4)e^{0.4p}}{(149 + e^{0.4p})^2} + \frac{45000}{149 + e^{0.4p}} = \frac{-45000(0.4pe^{0.4p}) + 45000(149 + e^{0.4p})}{(149 + e^{0.4p})^2}.$$

Since the denominator is never 0, $M'(p)$ exists for all p, so the only critical points of M occur when $-45000(0.4pe^{0.4p})+45000(149+e^{0.4p}) = 0$, i.e. when $149+e^{0.4p}-0.4pe^{0.4p} = 0$. Using a graphing calculator to find the horizontal (p–) intercept of this function, we find the critical point $p \approx 9.823$.

To see that this is the only critical point, let $f(p) = 149 + e^{0.4p} - 0.4pe^{0.4p}$. Then $f'(p) = 0.4e^{0.4p} - 0.4pe^{0.4p}(0.4) - 0.4e^{0.4p} = -0.16pe^{0.4p}$, so $f'(p) < 0$ for all $p > 0$. Hence f is a decreasing function on $[0, \infty)$. It follows that the only p–intercept of f for $p > 0$ is $p \approx 9.823$. Now $M'(9) > 0$ and $M'(10) < 0$, so M has a local maximum at $p \approx 9.823$ by the First Derivative Test. Since this is the only critical point, M has a global maximum at $p \approx 9.823$. $\boxed{\text{To maximize revenue, WFUH should charge \$9.82 per hour of usage.}}$ (Note that $M(9.82) > M(9.83)$, so the rounded answer is the best possible choice.)

c. [5 points] Suppose that WFUH initially decides to charge \$12.50 per hour of use. Note that $h(12.50)$ is approximately 151. Suppose the marginal cost to WFUH when 151 hours of wifi are being used is \$5 per hour of use. In order to increase its profit, should WFUH raise or lower the price it is charging per hour of use on NW flight 2337? Explain.

Solution: Marginal revenue (in dollars per hour of use) is given by $R'(h)$, where $R(h)$ is the revenue from h hours of use. Note that $R(h) = a + ph$, so $R'(h) = p'(h)h + p(h)$. So when $h = 151$, we have

$R'(151) = p'(151)(151)+p(151) = \dfrac{1}{h'(12.50)}(151)+12.50 \approx 7.50$. Therefore, the marginal revenue is about \$7.50 per hour of use, so since marginal cost is only \$5 per hour of use, the profit for WFUH will increase if the number of hours of usage increases from 151 hours. Since $h'(12.5) < 0$, the number of hours of use decreases when price increases from \$12.50 per hour. So, to increase the number of hours of usage (and hence increase profits), WFUH should $\boxed{\text{decrease}}$ the price it is charging per hour of use.

Alternate Approach: Marginal cost is $\dfrac{dC}{dh}$. By the Chain Rule, $\dfrac{dC}{dp} = \dfrac{dC}{dh} \cdot \dfrac{dh}{dp}$, so when $p = \$12.50$, $\dfrac{dC}{dp} = 5 \cdot h'(12.5)$. Computing, we find that $M'(12.5) < 5 \cdot h'(12.5)$. Hence, when price increases from $p = 12.50$, profit decreases, and when price decreases from $p = 12.5$, profit increases. Therefore, WFUH should decrease the price it is charging.