1. [10 points] Given below is a graph of a function $f(x)$ and a table for a function $g(x)$.

\[
\begin{array}{c|c|c|c|c|c}
  x & 0 & 1 & 2 & 3 & 4 \\
  g(x) & 4 & 3 & 1 & 2 & \frac{20}{3} \\
  g'(x) & -2 & -\frac{5}{2} & 1 & 2 & -\frac{3}{4} \\
\end{array}
\]

Give answers for the following or write “Does not exist.” No partial credit will be given.

i) $\frac{d}{dx} f(g(x))$ at $x = 0$  
By the chain rule $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$. At $x = 0$ we have

$$f'(g(0))g'(0) = f'(4) \cdot (-2) = 4.$$  

ii) $\frac{d}{dx} [f(x)g(x)]$ at $x = 2$  
By the product rule $\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$. At $x = 2$ we have

$$f'(2)g(2) + f(2)g'(2) = (2/3)(1) + (4/3)(1/2) = 4/3.$$  

iii) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right]$ at $x = 4$  
By the quotient rule $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$. At $x = 4$ we have

$$\frac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2} = \frac{-2(20/3) - 0(-1/3)}{(20/3)^2} = -3/10.$$  

iv) $\frac{d}{dx} [g(f(x))]$ at $x = 3$  
We know $g'(f(3)) = 1/2$, so for values of $y$ near $f(3)$, $g(y)$ looks like a line with slope $1/2$. So $g(f(x))$ “looks like” $\frac{1}{2} f(x) + b$ for some constant $b$, for $x$ near 3. Since $f(x)$ is “pointy” at $x = 3$, $\frac{1}{2} f(x) + b$ looks like a vertically compressed version of this pointy graph (near $x = 3$), which is still pointy. So $g(f(x))$ is also pointy at $x = 3$, hence not differentiable.

v) $f(g'(3))$  
By the table, $g'(3) = 3$, so $f(g'(3)) = f(3) = 2.$