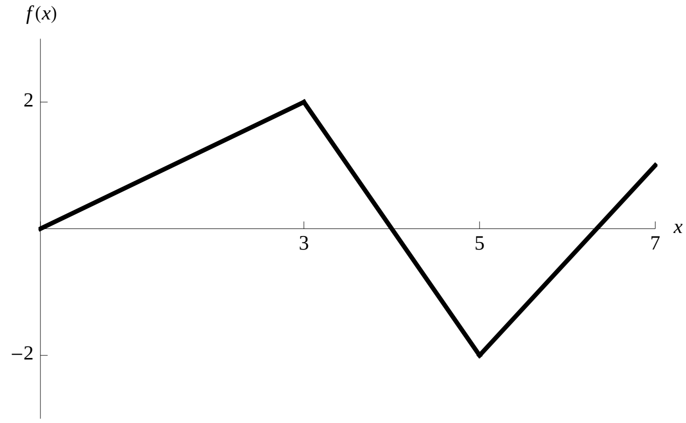
1. [10 points] Given below is a graph of a function $f(x)$ and a table for a function $g(x)$.



x	0	1	2	3	4
$g(x)$	4	3	1	2	$\frac{20}{3}$
$g'(x)$	-2	$-\frac{5}{2}$	$\frac{1}{2}$	3	$-\frac{1}{3}$

Give answers for the following or write "Does not exist." No partial credit will be given.

i) $\frac{d}{dx} f(g(x))$ at $x = 0$ By the chain rule $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$. At $x = 0$ we have

$$f'(g(0))g'(0) = f'(4) \cdot (-2) = 4.$$

ii) $\frac{d}{dx}[f(x)g(x)]$ at $x = 2$ By the product rule $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$. At $x = 2$ we have

$$f'(2)g(2) + f(2)g'(2) = (2/3)(1) + (4/3)(1/2) = 4/3.$$

iii) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x = 4$ By the quotient rule $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$. At $x = 4$ we have

$$\frac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2} = \frac{(-2)(20/3) - (0)(-1/3)}{(20/3)^2} = -3/10.$$

iv) $\frac{d}{dx}[g(f(x))]$ at $x = 3$ We know $g'(f(3)) = 1/2$, so for values of y near $f(3)$, $g(y)$ looks like a line with slope 1/2. So $g(f(x))$ "looks like" $\frac{1}{2}f(x)+b$ for some constant b, for x near 3. Since $f(x)$ is "pointy" at $x = 3$, $\frac{1}{2}f(x) + b$ looks like a vertically compressed version of this pointy graph (near $x = 3$), which is still pointy. So $g(f(x))$ is also pointy at $x = 3$, hence not differentiable.

v) $f(g'(3))$ By the table, $g'(3) = 3$, so $f(g'(3)) = f(3) = 2$.