**3**. [15 points] Answer "True" or "False" for each of the following, and **include a brief explanation of your answer.** A picture may be sufficient for an explanation, if appropriate. The functions $h, h', m$ and $m'$ referred to in the problem are all differentiable on their domain. The letters $a$ and $b$ represent constants.

i) If $y = h'(x)m(x) - h(x)m'(x)$, then $\frac{dy}{dx} = h''(x)m(x) - h(x)m''(x)$.

**[True]** False

*Solution:*

$$\frac{dy}{dx} = h''(x)m(x) + h'(x)m'(x) - h'(x)m'(x) - h(x)m''(x) = h''(x)m(x) - h(x)m''(x).$$

ii) If $m''(a) = 0$, then $m(x)$ has an inflection point at $x = a$.

True **[False]**

*Solution:* The function $f(x) = x^4$ has $f''(0) = 0$, but $f''$ is positive on either side of $x = 0$ so there is not an inflection point at $x = 0$.

iii) If $h''(x) > 0$ on the interval $[a, b]$ and $h(a) > h(b)$, then $h(a)$ is the absolute maximum value of $h(x)$ on $[a, b]$.

**[True]** False

*Solution:* Since $h$ is concave up for the entire interval, any critical points on the interval must be local minima. This means the maximum must occur at an endpoint (either $x = a$ or $x = b$) of the interval. The maximum value must be $h(a)$ because it is larger than $h(b)$.

iv) There exists a continuous function $f(x)$ which is not differentiable at $x = 0$ with a local maximum at $(0, 5)$.

**[True]** False

*Solution:* The function $f(x) = -|x| + 5$ has a local maximum at $(0, 5)$, but has a corner at this point, so it is not differentiable at $x = 0$.

v) The function $g(x) = e^{-(x-a)^2/b}$ has a local maximum at $x = b$.

True **[False]**

*Solution:* The derivative of $g$ is

$$g'(x) = -\frac{2(x-a)}{b}e^{-(x-a)^2/b}$$

so the only critical point of $g$ is at $x = a$. Therefore there cannot be a local max at $x = b$ since it is not a critical point.