5. [12 points] Suppose a curve in the plane is given by the equation

$$\sin(\pi xy) = y - 1.$$

a. [3 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

Solution: At $(1, 1)$, the right hand side is $\sin(\pi) = 0$ and the left hand side is $1 - 1 = 0$. Therefore the point is on the curve since the right and left hand sides are equal.

b. [5 points] Calculate $\dfrac{dy}{dx}$.

Solution: Taking the derivative with respect to x of the equation, we have

$$\pi \cos(\pi xy) \cdot (y + x\frac{dy}{dx}) = \frac{dy}{dx}.$$

Solving for $\frac{dy}{dx}$, we get

$$\frac{dy}{dx} = \frac{\pi y \cos(\pi xy)}{1 - \pi x \cos(\pi xy)}.$$

c. [4 points] Find the equation for the tangent line to the curve at the point $(1, 1)$.

Solution: The slope of the tangent line to the curve is

$$\frac{dy}{dx}(1, 1) = \frac{\pi \cos(\pi)}{1 - \pi \cos(\pi)} = \frac{-\pi}{1 + \pi}.$$

The equation for the tangent line is

$$y - 1 = \frac{-\pi}{1 + \pi}(x - 1).$$