

6. [10 points] Calvin is stuck in the desert, and he needs to build a cube out of cactus skins to hold various supplies. He wants his cube to have a volume of 8.1 cubic feet, but he needs to figure out the side length to cut the cactus skins the right size. He has forgotten his trusty calculator, so he decides to figure out the side length of his cube using calculus.

- a. [5 points] Find a local linearization of the function  $f(x) = (x + 8)^{1/3}$  at  $x = 0$ .

*Solution:* The derivative is  $f'(x) = \frac{1}{3}(x + 8)^{-2/3}$ . To find the local linearization we compute  $f'(0) = \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$  and  $f(0) = 2$ . The equation for the tangent line to  $f$  at  $x = 0$  is  $y - 2 = \frac{1}{12}x$ . So the local linearization of  $f$  near  $x = 0$  is

$$L(x) = \frac{1}{12}x + 2.$$

- b. [3 points] Use your linearization to approximate  $(8.1)^{1/3}$ .

*Solution:* We need to approximate  $(8.1)^{1/3} = f(0.1)$ . According to our local linearization,

$$f(0.1) \approx L(0.1) = \frac{1}{12}(0.1) + 2 = \frac{241}{120}.$$

- c. [2 points] Should your approximation from part **b.** be an over-estimate or an under-estimate? Why?

*Solution:*

The second derivative of  $f$  is  $f''(x) = -\frac{2}{9}(x + 8)^{-5/3}$ . For values of  $x$  near 0, the second derivative will be negative which means  $f$  is concave down near 0. This means our estimate is an overestimate.