7. [15 points] Suppose a is a positive constant and

$$f(x) = 2x^3 - 3ax^2.$$

a. [10 points] Find the absolute maximum and minimum values of $f(x)$ on the closed interval $[-a, \frac{3}{2}a]$. Specify all x values where the maximum and minimum value are achieved.

Solution: Seeking critical points, we take the derivative of f and set it equal to zero.

$$f'(x) = 6x^2 - 6ax = 6x(x-a) = 0.$$

Using this equation we find the critical points to be $x = 0, a$. Now we put the critical points and the endpoints of the interval back into the orginal function and compare the values. We compute $f(-a) = -5a^3,\ f(0) = 0,\ f(a) = -a^3,\ f(\frac{3}{2}a) = 0$.
This means the absolute max of f on this interval is 0 and this value is achieved at $x = 0, \frac{3}{2}a$. The absolute min is $-5a^3$ and this value is achieved at $x = -a$.

b. [5 points] Find all inflection points of $f(x)$.

Solution: Seeking inflection points, we find $f''(x) = 12x - 6a$. Setting this equal to zero we find $x = \frac{a}{2}$. To verify this is an inflection point we test $f''(0) = -6a < 0$ and $f''(a) = 6a > 0$. This means f'' changes sign at $x = \frac{a}{2}$, so it is an inflection point.