1. [9 points] Let $U = f(t)$ give the number of Facebook users in millions in year t. Suppose $f(2005) = 5.5$ and $f'(2005) = 4.9$. For this problem assume that $f(t)$ is strictly increasing.

a. [4 points] Find and interpret, in practical terms, $f^{-1}(5.5)$.

Solution:
The year in which there were 5.5 million Facebook users was 2005.

$f^{-1}(5.5) =$ 2005**(year)**

b. [5 points] Showing work, evaluate $(f^{-1})'(5.5)$. Interpret your answer in practical terms.

Solution:
$(f^{-1})'(5.5) = 1/(f'(f^{-1}(5.5)) = 1/f'(2005) = 1/4.9$.

When the number of Facebook users was 5.5 million, in 1/4.9 (∼0.2) years the number of Facebook users will have increased by approximately 1 million users.

$(f^{-1})'(5.5) =$ 1/4.9 **years per million users**

2. [8 points] Recall the function $T(x)$ that took the number of followers (in millions) of a Twitter user and returned a value from 0 to 10 called the user's Twitter celebrity index. The derivative of $T(x)$ is given by the function

$$T'(x) = \frac{1532.5 \cdot (0.6)^x}{(5 + 60(0.6)^x)^2}.$$

a. [4 points] If $T(3) = 1.56$, compute the local linearization of $T(x)$ near $x = 3$.

Solution: The formula for the local linearization for a function $f(x)$ near a point $x = a$ is

$$L(x) = f(a) + f'(a)(x - a).$$

To use this formula for $T(x)$, we will need to find $T'(3) = \frac{1532.5 \cdot (0.6)^3}{(5+60(0.6)^3)^2} \approx 1.0262$. Then, the local linearization is $L(x) = 1.56 + 1.0262(x - 3)$.

b. [4 points] Use your expression from (a) to approximate the Twitter celebrity index of a celebrity with 3.2 million followers.

Solution: $L(3.2) = 1.56 + 1.0262(3.2 - 3) = 1.7652$ (TCI).