6. [13 points] Let $f(v)$ be the gas consumption (in liters/km) of a car going at velocity v (in km/hr). In other words, $f(v)$ tells you how many liters of gas the car uses to go one kilometer, if it is going at velocity v. You are told that

$$f(90) = 0.08 \text{ and } f'(90) = 0.0008.$$

a. [5 points] Let $g(v)$ be the distance the same car goes on one liter of gas at velocity v. What is the relationship between $f(v)$ and $g(v)$? Find $g(90)$ and $g'(90)$.

Solution: The function $f(v)$ is consumption in L/km and $g(v)$ is efficiency in km/L, so the relationship between $f(v)$ and $g(v)$ is

$$g(v) = 1/f(v) = [f(v)]^{-1}.$$

This means $g(90) = 1/f(90) = 1/0.08 = 12.5$ km/L. The derivative of $g(v)$ is

$$g'(v) = -[f(v)]^{-2} f'(v).$$

so $g'(90) = -[f(90)]^{-2} f'(90) = -(0.08)^{-2}(0.0008) = -0.125$ km/L per km/h.

b. [5 points] Let $h(v)$ be the gas consumption in liters per hour. In other words, $h(v)$ tells you how many liters of gas the car uses in one hour if the car is going at velocity v. What is the relationship between $h(v)$ and $f(v)$? Find $h(90)$ and $h'(90)$.

Solution: Since $f(v)$ is consumption in L/km and v is velocity in km/h, the function $h(v)$ must be the product of v and $f(v)$, in L/h.

$$h(v) = vf(v).$$

This means $h(90) = 90f(90) = 7.2$ L/h. The derivative of $h(v)$ is

$$h'(v) = f(v) + vf'(v)$$

so $h'(90) = f(90) + 90f'(90) = 0.152$ L/h per km/h.

c. [3 points] How would you explain the practical meaning of $g'(90)$ to a driver who knows no calculus?

Solution: The value of $g'(90)$ is -0.125 km/L per km/h. In practical terms this means: "When the car increases speed from 90 to 91 km/h, the fuel efficiency of the car decreases by about 0.125 km/L."