7. [16 points] Let $f(x) = \ln(x)$. Use the table of values below for $g(x)$ and $g'(x)$ to answer the following questions.

x	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

a. [4 points] If $F(x) = f(g(x))$, find $F'(4)$.

Solution:

$$F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x))g'(x) \text{ by the Chain Rule}$$

Then, to find $F'(4)$, we plug 4 in for x and take the correct values from the table:

$$F'(4) = f'(g(4))g'(4) = \frac{g'(4)}{g(4)} = \frac{2}{6} = \frac{1}{3}.$$

b. [4 points] If $G(x) = g^{-1}(x)$, find $G'(4)$.

Solution:

$$G(x) = g^{-1}(x) \quad \Rightarrow \quad g(g^{-1}(x)) = x \text{ and by the Chain Rule } \left(g^{-1}\right)'(x) = \frac{1}{g'(g^{-1}(x))}$$

Then, to find $G'(4)$, we plug 4 in for x and take the correct values from the table:

$$G'(4) = \frac{1}{g'(g^{-1}(4))} = \frac{1}{g'(3)} = \frac{1}{3}.$$

c. [4 points] If $H(x) = \tan(g(x))$, find $H'(3)$.

Solution: To find the derivative of H, you can either recall the derivative of tangent from your notecard, or use the quotient rule.

$$H(x) = \tan(g(x)) \quad \Rightarrow \quad H'(x) = \frac{g'(x)}{\cos^2(g(x))}.$$

Then, to find $H'(3)$, we plug 3 in for x and take the correct values from the table:

$$H'(3) = \frac{g'(3)}{\cos^2(g(3))} = \frac{3}{\cos^2(4)} \approx 7.0217.$$

d. [4 points] If $E(x) = e^{f(x)g(x)}$, find $E'(2)$.

Solution:

$$E(x) = e^{f(x)g(x)} \quad \Rightarrow \quad E'(x) = e^{f(x)g(x)}\left(f'(x)g(x) + f(x)g'(x)\right)$$

$$\text{Since } f(x) = \ln(x), \qquad E'(z) = e^{\ln(x)g(x)}\left(\frac{g(x)}{x} + \ln(x)g'(x)\right).$$

Then, to find $E'(2)$, we plug 2 in for x and take the correct values from the table:

$$E'(2) = e^{\ln(2)g(2)}\left(\frac{g(2)}{2} + \ln(2)g'(2)\right) = e^{\ln(2)}\left(\frac{1}{2} + 5\ln(2)\right) \approx 7.9315.$$