8. [12 points] The equation $(x^2+y^2)^2 = 4x^2y$ describes a two-petaled rose curve.

a. [2 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

Solution: At the point $(x, y) = (1, 1)$,

$$(x^2+y^2)^2 = (1^2+1^2)^2 = 4 = 4(1)^2(1) = 4x^2y.$$

b. [7 points] Calculate dy/dx at $(x, y) = (1, 1)$.

Solution: Differentiating both sides of the equation for the curve with respect to x we have

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 4\left(2xy + x^2\frac{dy}{dx}\right).$$

At the point $(x, y) = (1, 1)$ this equation becomes

$$2(1^2+1^2)\left(2(1)+2(1)\frac{dy}{dx}\right) = 4\left(2(1)(1) + (1)^2\frac{dy}{dx}\right).$$

Simplifying, we have $4(2+2\frac{dy}{dx}) = 8 + 4\frac{dy}{dx}$. This gives us that $\frac{dy}{dx} = 0$ at $(x, y) = (1, 1)$.

c. [3 points] Find the equation of the tangent line to the rose curve at the point $(x, y) = (1, 1)$.

Solution: Using point slope form, the tangent line is $y - 1 = 0(x - 1)$. Simplifying, we have that the tangent line to the rose curve at $(x, y) = (1, 1)$ is $y = 1$.