8. [12 points] The equation \((x^2 + y^2)^2 = 4x^2y\) describes a two-petaled rose curve.

a. [2 points] Verify that the point \((x, y) = (1, 1)\) is on the curve.

Solution: At the point \((x, y) = (1, 1)\),
\[
(x^2 + y^2)^2 = (1^2 + 1^2)^2 = 4 = 4(1)^2(1) = 4x^2y.
\]

b. [7 points] Calculate \(dy/dx\) at \((x, y) = (1, 1)\).

Solution: Differentiating both sides of the equation for the curve with respect to \(x\) we have
\[
2(x^2 + y^2) \left( 2x + 2y \frac{dy}{dx} \right) = 4 \left( 2xy + x^2 \frac{dy}{dx} \right).
\]
At the point \((x, y) = (1, 1)\) this equation becomes
\[
2(1^2 + 1^2) \left( 2(1) + 2(1) \frac{dy}{dx} \right) = 4 \left( 2(1)(1) + (1)^2 \frac{dy}{dx} \right).
\]
Simplifying, we have \(4(2 + 2 \frac{dy}{dx}) = 8 + 4 \frac{dy}{dx}\). This gives us that \(\frac{dy}{dx} = 0\) at \((x, y) = (1, 1)\).

c. [3 points] Find the equation of the tangent line to the rose curve at the point \((x, y) = (1, 1)\).

Solution: Using point slope form, the tangent line is \(y - 1 = 0(x - 1)\). Simplifying, we have that the tangent line to the rose curve at \((x, y) = (1, 1)\) is \(y = 1\).