1. [12 points] The following questions relate to the implicit curve $2x^2 + 4x - x^2y^2 + 3y^4 = -1$.

a. [6 points] Calculate $\frac{dy}{dx}$.

Solution: Differentiating both sides with respect to x, we get

$$4x + 4 - 2xy^2 - 2x^2y\,\frac{dy}{dx} + 12y^3\,\frac{dy}{dx} = 0.$$

Moving all terms with no $\frac{dy}{dx}$ to the other side and factoring out $\frac{dy}{dx}$ gives us

$$\frac{dy}{dx}(12y^3 - 2x^2y) = 2xy^2 - 4x - 4.$$

So

$$\frac{dy}{dx} = \frac{2xy^2 - 4x - 4}{12y^3 - 2x^2y} = \frac{xy^2 - 2x - 2}{6y^3 - x^2y}.$$

b. [2 points] Q is the only point on the curve that has a y-coordinate of 1. Find the x-coordinate of Q.

Solution: Plugging $y = 1$ into the equation for the curve gives us

$$2x^2 + 4x - x^2 + 3 = -1.$$

Moving all the terms to the left, we get

$$x^2 + 4x + 4 = 0.$$

This factors as $(x + 2)^2 = 0$, so $x = -2$.

c. [4 points] Find the equation of the tangent line to the curve at Q.

Solution: To find the slope, we plug in $x = -2$ and $y = 1$ to $\frac{dy}{dx}$.

$$\text{slope} = \frac{-2 + 4 - 2}{6 - 4} = 0.$$

Thus, the tangent line is the horizontal line passing through Q, which has equation $y = 1$.