3. [8 points] Jason has a 50 ft by 30 ft house and wants to enclose his yard with a fence to keep his dogs in, as shown below. On the south side of his yard, he plans for the fence to be extra tall to shade his yard from the sun. Note that the fence does not extend around the sides of Jason's house. The extra tall fence (thick dashed line) costs $15 per foot, and the rest of the fence (thin dashed line) costs $5 per foot. Jason is going to spend $4500 on his fence.
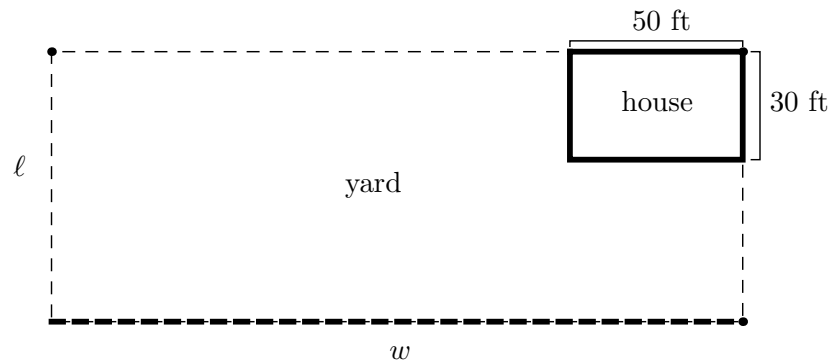
a. [5 points] ℓ is the length of the fenced in yard, and w is the width, as shown above. Write a formula for ℓ in terms of w. Your formula should not involve any other variables.

> *Solution:* $4500 = 15w + 5(\ell + (w - 50) + (\ell - 30)) = 20w + 10\ell - 400$. So $\ell = 490 - 2w$.

b. [3 points] Write a formula for the total area A of the fenced yard (not including the house), in terms of w. Your answer should not include ℓ. (This is the equation Jason would use to find the values of w and ℓ maximizing the area he can enclose. You should **not** do the optimization in this case.)

> *Solution:* $A = w\ell - 50 \cdot 30 = w\ell - 1500$. Substituting in $\ell = 490 - 2w$, this becomes $A = w(490 - 2w) - 1500$.