1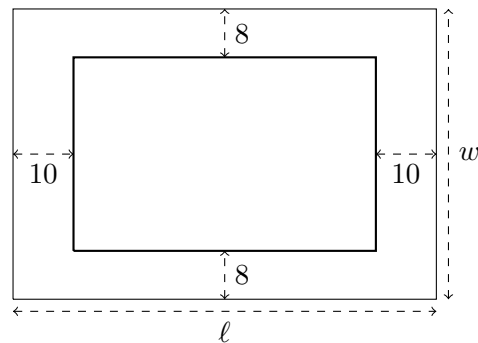. [7 points] Liam wants to build a rectangular swimming pool behind his new house. The pool will have an area of 1600 square feet. He will have 8-foot wide decks on two sides of the pool and 10-foot wide decks on the other two sides of the pool (see the diagram below).

a. [4 points] Let ℓ and w be the length and width (in feet) of the pool area including the decks as shown in the diagram. Write a formula for ℓ in terms of w.

> *Solution:* The area of the pool needs to be 1600 sq ft, so
>
> $$(\ell - 2(10))(w - 2(8)) = 1600$$
>
> Solving this for ℓ gives
> $$\ell = \frac{1600}{w - 16} + 20$$

b. [3 points] Write a formula for the function $A(w)$ which gives the total area (in square feet) of the pool **and** the decks in terms of only the width w. Your formula should not include the variable ℓ. (This is the function Liam would minimize in order to find the minimum area that his pool and deck will take up in his yard. You do not need to do the optimization in this case.)

> *Solution:* The pool and decks together make a rectangle of length ℓ and width w. The area A of the rectangle is $A = \ell w$. Substituting the formula from part (a) gives
>
> $$A(w) = \left(\frac{1600}{w - 16} + 20 \right) w$$