**2.** [6 points] Given the implicit curve  $y^2 = \cos(xy) - 3x$ , find  $\frac{dy}{dx}$ .

Solution: Using implicit differentiation, we get

$$2y\frac{dy}{dx} = -\sin(xy)\left(y + x\frac{dy}{dx}\right) - 3$$

Solving for  $\frac{dy}{dx}$  gives

$$2y\frac{dy}{dx} = -y\sin(xy) - x\sin(xy)\frac{dy}{dx} - 3$$
$$2y\frac{dy}{dx} + x\sin(xy)\frac{dy}{dx} = -y\sin(xy) - 3$$
$$\frac{dy}{dx} = \frac{-y\sin(xy) - 3}{2y + x\sin(xy)}$$

- **3.** [9 points] This problem concerns the function  $f(x) = -x 3e^{4x}$ .
  - **a.** [3 points] Show that the function f is invertible.

Solution: We have  $f'(x) = -1 - 12e^{-4x}$  which is negative for all values of x. This means that f is a strictly decreasing function. Since f is strictly decreasing, it never takes the same value twice so f is invertible.

- **b.** [2 points] Find  $f^{-1}(-3)$ . You do not need to show any work. Solution:  $f^{-1}(-3) = 0$  because f(0) = -3.
- c. [4 points] Evaluate  $(f^{-1})'(-3)$ . Show all of your work.

Solution: Using the formula for the derivative of an inverse function, we get

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))} = \frac{1}{f'(0)}$$

Since  $f'(x) = -1 - 12e^{-4x}$  we have f'(0) = -13 and so

$$(f^{-1})'(-3) = -\frac{1}{13}$$