2. [6 points] Given the implicit curve $y^{2}=\cos (x y)-3 x$, find $\frac{d y}{d x}$.

Solution: Using implicit differentiation, we get

$$
2 y \frac{d y}{d x}=-\sin (x y)\left(y+x \frac{d y}{d x}\right)-3
$$

Solving for $\frac{d y}{d x}$ gives

$$
\begin{aligned}
2 y \frac{d y}{d x} & =-y \sin (x y)-x \sin (x y) \frac{d y}{d x}-3 \\
2 y \frac{d y}{d x}+x \sin (x y) \frac{d y}{d x} & =-y \sin (x y)-3 \\
\frac{d y}{d x} & =\frac{-y \sin (x y)-3}{2 y+x \sin (x y)}
\end{aligned}
$$

3. [9 points] This problem concerns the function $f(x)=-x-3 e^{4 x}$.
a. [3 points] Show that the function $f$ is invertible.

Solution: We have $f^{\prime}(x)=-1-12 e^{-4 x}$ which is negative for all values of $x$. This means that $f$ is a strictly decreasing function. Since $f$ is strictly decreasing, it never takes the same value twice so $f$ is invertible.
b. [2 points] Find $f^{-1}(-3)$. You do not need to show any work.

Solution: $\quad f^{-1}(-3)=0$ because $f(0)=-3$.
c. [4 points] Evaluate $\left(f^{-1}\right)^{\prime}(-3)$. Show all of your work.

Solution: Using the formula for the derivative of an inverse function, we get

$$
\left(f^{-1}\right)^{\prime}(-3)=\frac{1}{f^{\prime}\left(f^{-1}(-3)\right)}=\frac{1}{f^{\prime}(0)}
$$

Since $f^{\prime}(x)=-1-12 e^{-4 x}$ we have $f^{\prime}(0)=-13$ and so

$$
\left(f^{-1}\right)^{\prime}(-3)=-\frac{1}{13}
$$

