2. [6 points] Given the implicit curve $y^2 = \cos(xy) - 3x$, find $\dfrac{dy}{dx}$.

Solution: Using implicit differentiation, we get

$$2y\frac{dy}{dx} = -\sin(xy)\left(y + x\frac{dy}{dx}\right) - 3$$

Solving for $\dfrac{dy}{dx}$ gives

$$\begin{aligned} 2y\frac{dy}{dx} &= -y\sin(xy) - x\sin(xy)\frac{dy}{dx} - 3 \\ 2y\frac{dy}{dx} + x\sin(xy)\frac{dy}{dx} &= -y\sin(xy) - 3 \\ \frac{dy}{dx} &= \frac{-y\sin(xy) - 3}{2y + x\sin(xy)} \end{aligned}$$

3. [9 points] This problem concerns the function $f(x) = -x - 3e^{4x}$.

a. [3 points] Show that the function f is invertible.

Solution: We have $f'(x) = -1 - 12e^{-4x}$ which is negative for all values of x. This means that f is a strictly decreasing function. Since f is strictly decreasing, it never takes the same value twice so f is invertible.

b. [2 points] Find $f^{-1}(-3)$. You do not need to show any work.

Solution: $f^{-1}(-3) = 0$ because $f(0) = -3$.

c. [4 points] Evaluate $(f^{-1})'(-3)$. Show all of your work.

Solution: Using the formula for the derivative of an inverse function, we get

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))} = \frac{1}{f'(0)}$$

Since $f'(x) = -1 - 12e^{-4x}$ we have $f'(0) = -13$ and so

$$(f^{-1})'(-3) = -\frac{1}{13}$$