2. [6 points] Given the implicit curve \( y^2 = \cos(xy) - 3x \), find \( \frac{dy}{dx} \).

Solution: Using implicit differentiation, we get

\[
2y \frac{dy}{dx} = -\sin(xy) \left( y + x \frac{dy}{dx} \right) - 3
\]

Solving for \( \frac{dy}{dx} \) gives

\[
2y \frac{dy}{dx} = -y \sin(xy) - x \sin(xy) \frac{dy}{dx} - 3
\]

\[
2y \frac{dy}{dx} + x \sin(xy) \frac{dy}{dx} = -y \sin(xy) - 3
\]

\[
\frac{dy}{dx} = \frac{-y \sin(xy) - 3}{2y + x \sin(xy)}
\]

3. [9 points] This problem concerns the function \( f(x) = -x - 3e^{4x} \).

a. [3 points] Show that the function \( f \) is invertible.

Solution: We have \( f'(x) = -1 - 12e^{-4x} \) which is negative for all values of \( x \). This means that \( f \) is a strictly decreasing function. Since \( f \) is strictly decreasing, it never takes the same value twice so \( f \) is invertible.

b. [2 points] Find \( f^{-1}(-3) \). You do not need to show any work.

Solution: \( f^{-1}(-3) = 0 \) because \( f(0) = -3 \).

c. [4 points] Evaluate \( (f^{-1})'(-3) \). Show all of your work.

Solution: Using the formula for the derivative of an inverse function, we get

\[
(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))} = \frac{1}{f'(0)}
\]

Since \( f'(x) = -1 - 12e^{-4x} \) we have \( f'(0) = -13 \) and so

\[
(f^{-1})'(-3) = \frac{1}{13}
\]