4. [17 points] The function $g(x)$ is continuous on the interval $0<x<8$. The graph of $g^{\prime}(x)$, the derivative of $g(x)$, is shown below.

a. [6 points] List the $x$-coordinates of the critical points of the function $g(x)$ and state whether each is a local maximum, local minimum, or neither. You do not need to justify your answers.

| Solution: | $x=0.5$ | $x=3$ | $x=4$ | $x=6$ |
| :--- | :--- | :--- | :--- | :--- |
|  | local minimum | local maximum | local minimum | neither |

b. [3 points] List the $x$-coordinates of the inflection points of the function $g(x)$. You do not need to justify your answers.
Solution: $\quad x=2, x=6$
c. [3 points] Suppose that $g(1)=8$. Write an equation for the best linear approximation to $g(x)$ at $x=1$.

$$
g(x) \approx \frac{10(x-1)+8}{}
$$

d. [2 points] Use your approximation from part (c) to estimate $g(1.05)$.

Solution: $\quad g(1.05) \approx 10(1.05-1)+8=8.5$
e. [3 points] Is your estimate for $g(1.05)$ an overestimate or an underestimate? Explain.

Solution: We see from the graph that $g^{\prime}(x)$ is increasing at $x=1$, so $g(x)$ is concave up at $x=1$. Because the graph of $g(x)$ is concave up at $x=1$, the tangent line is below the curve so our estimate is an underestimate.

