5. [6 points] For each of the following statements, circle True if the statement is always true and circle False otherwise. No justification is necessary.

a. [2 points] If the function $f(x)$ is continuous on the interval $(0, 100)$, then $f(x)$ has a global maximum and a global minimum on that interval.

True **[False]**

b. [2 points] If $f(x)$ is a differentiable function with a critical point at $x = c$, then the function $g(x) = e^{f(x)}$ also has a critical point at $x = c$.

[True] False

c. [2 points] If $f'(x)$ is continuous and $f'(x) \neq 0$ for all x, then $f(0) \neq f(5)$.

[True] False

6. [8 points] This problem concerns the implicit curve

$$x^2 + xy + y^2 = 7$$

for which

$$\frac{dy}{dx} = \frac{-y - 2x}{x + 2y}.$$

a. [3 points] Find an equation for the tangent line to the curve at the point $(1, 2)$.

Solution:

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-2 - 2(1)}{1 + 2(2)} = -\frac{4}{5}$$

So the tangent line at $(1, 2)$ is $y = -\frac{4}{5}(x - 1) + 2$.

b. [5 points] Find the x- and y-coordinates of all points on the curve at which the tangent line is vertical.

Solution: If the tangent line is vertical, the slope will be undefined. The derivative $\frac{dy}{dx}$ is undefined when $x + 2y = 0$ which means $x = -2y$. Plugging this into the equation for the curve, we get

$$\begin{aligned}(-2y)^2 + (-2y)y + y^2 &= 7\\ y^2 &= \frac{7}{3}\\ y &= \pm\sqrt{\frac{7}{3}}\end{aligned}$$

Since $x = -2y$ this gives two points $\left(2\sqrt{\frac{7}{3}}, -\sqrt{\frac{7}{3}}\right)$ and $\left(-2\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}}\right)$.