5. [6 points] For each of the following statements, circle True if the statement is always true and circle False otherwise. No justification is necessary.

a. [2 points] If the function \( f(x) \) is continuous on the interval \((0, 100)\), then \( f(x) \) has a global maximum and a global minimum on that interval.

   True \hspace{1cm} False

b. [2 points] If \( f(x) \) is a differentiable function with a critical point at \( x = c \), then the function \( g(x) = e^{f(x)} \) also has a critical point at \( x = c \).

   True \hspace{1cm} False

c. [2 points] If \( f'(x) \) is continuous and \( f'(x) \neq 0 \) for all \( x \), then \( f(0) \neq f(5) \).

   True \hspace{1cm} False

6. [8 points] This problem concerns the implicit curve 
\[
x^2 + xy + y^2 = 7
\]
for which
\[
\frac{dy}{dx} = \frac{-y - 2x}{x + 2y}.
\]

a. [3 points] Find an equation for the tangent line to the curve at the point \((1, 2)\).

   Solution: \[
   \left. \frac{dy}{dx} \right|_{(1,2)} = \frac{-2 - 2(1)}{1 + 2(2)} = -\frac{4}{5}
   \]
   So the tangent line at \((1, 2)\) is \[y = -\frac{4}{5}(x - 1) + 2\].

b. [5 points] Find the \( x \)- and \( y \)-coordinates of all points on the curve at which the tangent line is vertical.

   Solution: If the tangent line is vertical, the slope will be undefined. The derivative \( \frac{dy}{dx} \) is undefined when \( x + 2y = 0 \) which means \( x = -2y \). Plugging this into the equation for the curve, we get
\[
(-2y)^2 + (-2y)y + y^2 = 7
\]
\[
y^2 = \frac{7}{3}
\]
\[
y = \pm \sqrt{\frac{7}{3}}
\]
Since \( x = -2y \) this gives two points \((2\sqrt{\frac{7}{3}}, -\sqrt{\frac{7}{3}})\) and \((-2\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}})\).