8. [12 points] For Thanksgiving, Bert is trying to make a festive feast table using fall-colored cloth and other accessories. The cloth costs \$0.25 per square foot and the accessories are \$0.50 each. He decides the impact of the festive table, I, is a function of the number of square feet of cloth, c, that he uses and the number of accessories, a, that he uses. This relationship is given by

$$I = c\left(\frac{1}{2}a - 3\right)^2.$$

Bert has a total budget of \$9 for the cloth and accessories.

a. [2 points] Write an equation which expresses that the total cost of the cloth plus the accessories for the festive table is \$9.

Solution:

$$0.25c + 0.5a = 9$$

b. [10 points] Use your answer from **(a)** to find the maximum impact of the festive table that is possible for \$9, as well as how many accessories and how much cloth is needed to achieve the maximum impact. Be sure to show your answer is indeed the maximum.

Solution: From part (a) we get

$$a = 18 - 0.5c$$

Plugging this into the formula for impact gives

$$I = c\left(\frac{1}{2}(18 - 0.5c) - 3\right)^2 = c\left(6 - \frac{1}{4}c\right)^2$$

We need to maximize I on the domain $0 \le c \le 36$. Taking the derivative with respect to c gives

$$\frac{dI}{dc} = \left(6 - \frac{1}{4}c\right)^2 + c\left(2\left(6 - \frac{1}{4}c\right)\left(-\frac{1}{4}\right)\right) = \left(6 - \frac{1}{4}c\right)\left(6 - \frac{3}{4}c\right)$$

Then $\dfrac{dI}{dc} = 0$ when $c = 24$ or $c = 8$. We test the critical points and the endpoints:

$$\begin{array}{ll} c = 0 & I = 0 \\ c = 8 & I = 128 \\ c = 24 & I = 0 \\ c = 36 & I = 324 \end{array}$$

and find the the maximum impact $I = 324$ occurs if $c = 36$. Using $a = 18 - 0.5c$ we find that $a = 0$ at this point.

maximum impact: $I =$ **324**

$c =$ **36**

$a =$ **0**