8. [12 points] For Thanksgiving, Bert is trying to make a festive feast table using fall-colored cloth and other accessories. The cloth costs $0.25 per square foot and the accessories are $0.50 each. He decides the impact of the festive table, \( I \), is a function of the number of square feet of cloth, \( c \), that he uses and the number of accessories, \( a \), that he uses. This relationship is given by

\[
I = c \left( \frac{1}{2}a - 3 \right)^2.
\]

Bert has a total budget of $9 for the cloth and accessories.

a. [2 points] Write an equation which expresses that the total cost of the cloth plus the accessories for the festive table is $9.

Solution:

\[
0.25c + 0.5a = 9
\]

b. [10 points] Use your answer from (a) to find the maximum impact of the festive table that is possible for $9, as well as how many accessories and how much cloth is needed to achieve the maximum impact. Be sure to show your answer is indeed the maximum.

Solution: From part (a) we get

\[
a = 18 - 0.5c
\]

Plugging this into the formula for impact gives

\[
I = c \left( \frac{1}{2}(18 - 0.5c) - 3 \right)^2 = c \left( 6 - \frac{1}{4}c \right)^2
\]

We need to maximize \( I \) on the domain \( 0 \leq c \leq 36 \). Taking the derivative with respect to \( c \) gives

\[
\frac{dI}{dc} = \left( 6 - \frac{1}{4}c \right)^2 + c \left( 2 \left( 6 - \frac{1}{4}c \right) \left( -\frac{1}{4} \right) \right) = \left( 6 - \frac{1}{4}c \right) \left( 6 - \frac{3}{4}c \right)
\]

Then \( \frac{dI}{dc} = 0 \) when \( c = 24 \) or \( c = 8 \). We test the critical points and the endpoints:

\[
\begin{align*}
c & = 0 & I & = 0 \\
c & = 8 & I & = 128 \\
c & = 24 & I & = 0 \\
c & = 36 & I & = 324
\end{align*}
\]

and find the the maximum impact\( I = 324 \) occurs if \( c = 36 \). Using \( a = 18 - 0.5c \) we find that \( a = 0 \) at this point.

maximum impact: \( I = \boxed{324} \)

\[
c = \boxed{36}
\]

\[
a = \boxed{0}
\]