

9. [10 points] Our friend Oren, the Math 115 student, wants to minimize how long it will take him to complete his upcoming web homework assignment. Before starting the assignment, he buys a cup of tea containing 55 milligrams of caffeine.

Let  $H(x)$  be the number of minutes it will take Oren to complete tonight's assignment if he consumes  $x$  milligrams of caffeine. For  $10 \leq x \leq 55$

$$H(x) = \frac{1}{120}x^2 - \frac{4}{3}x + 20 \ln(x).$$

Instead of immediately starting the assignment, he solves a calculus problem to determine how much caffeine he should consume.

- a. [8 points] Find all the values of  $x$  at which  $H(x)$  attains global extrema on the interval  $10 \leq x \leq 55$ . Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

(For each answer blank below, write NONE in the answer blank if appropriate.)

**Answer:** global min(s) at  $x =$  \_\_\_\_\_

**Answer:** global max(es) at  $x =$  \_\_\_\_\_

- b. [2 points] Assuming Oren consumes at least 10 milligrams and at most 55 milligrams of caffeine, what is the shortest amount of time it could take for him to finish his assignment? *Remember to include units.*

**Answer:** \_\_\_\_\_