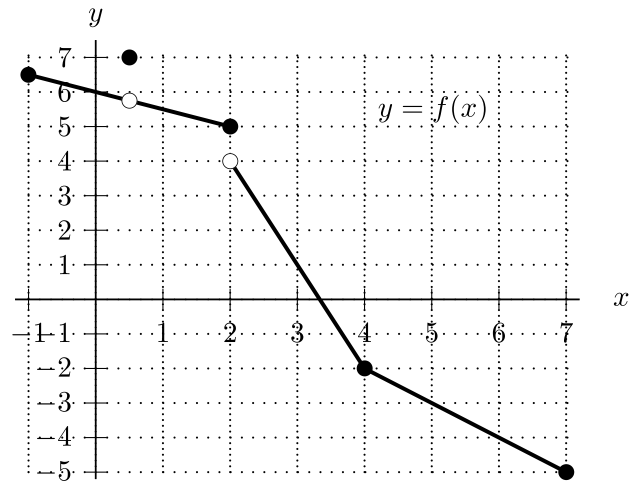
2. [11 points]

Shown to the right is the graph of a function $f(x)$.



Note that you are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

Find each of the following values. If the value does not exist, write DOES NOT EXIST.

a. [3 points] Let $h(x) = f(3x+1)$. Find $h'(1)$.

Solution: Since the graph $y = h(x)$ corresponds to the graph of $y = f(x)$ shifted left 1 unit and then horizontally compressed by a factor of $1/3$, $h(x)$ has a "sharp corner" at $x = 1$ so is not differentiable there.

Answer: $h'(1) =$ DOES NOT EXIST

b. [3 points] Let $k(x) = e^{f'(x)}$. Find $k'(6)$.

Solution: By the chain rule, $k'(x) = e^{f'(x)}f''(x)$. So $k'(6) = e^{f'(6)}f''(6) = e^{-1}(0) = 0$.

Answer: $k'(6) =$ 0

c. [2 points] Find $(f^{-1})'(0)$.

Solution: By the formula for the derivative of an inverse,

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(10/3)} = \frac{1}{-3}.$$

Answer: $(f^{-1})'(0) =$ $-1/3$

d. [3 points] Let $j(x) = \dfrac{f(2x+1)}{x+1}$. Find $j'(1)$.

Solution: Applying the quotient and chain rules, we find that

$$j'(x) = \frac{2f'(2x+1)(x+1) - f(2x+1)(1)}{(x+1)^2}.$$

Thus,

$$j'(1) = \frac{2f'(3)(2) - f(3)}{2^2} = \frac{(2)(-3)(2) - (1)}{4} = \frac{-13}{4}.$$

Answer: $j'(1) =$ $-13/4$