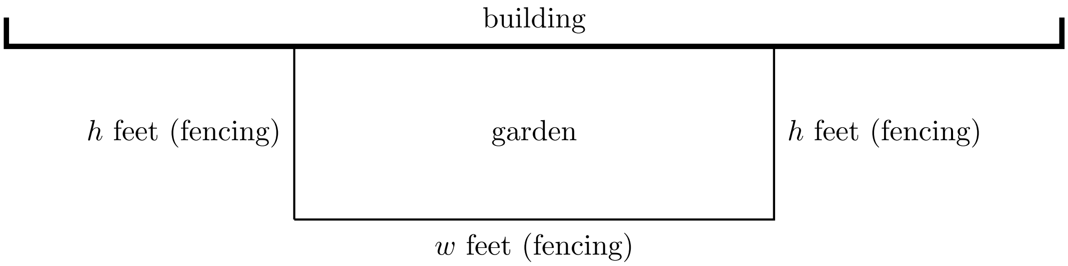
4. [12 points] Researchers are constructing a rectangular garden adjacent to their building. The garden will be bounded by the building on one side and by a fence on the other three sides. (See diagram below.) The fencing will cost them \$5 per linear foot. In addition, they will also need topsoil to cover the entire area of the garden. The topsoil will cost \$4 per square foot of the garden's area.
Assume the building is wider than any garden the researchers could afford to build.



a. [5 points] Suppose the garden is w feet wide and extends h feet from the building, as shown in the diagram above. Assume it costs the researchers a total of \$250 for the fencing and topsoil to construct this garden. Find a formula for w in terms of h.

Solution: A garden of these dimensions will require $2h+w$ feet of fencing and hw square feet of ground covered by topsoil. Thus,

$$250 = 5(2h+w) + 4hw\,.$$

Solving for w we find

$$w = \frac{250-10h}{4h+5}\,.$$

Answer: $w = \frac{250-10h}{4h+5}$

b. [3 points] Let $A(h)$ be the total area (in square feet) of the garden if it costs \$250 and extends h feet from the building, as shown above. Find a formula for the function $A(h)$. The variable w should not appear in your answer.

(Note that $A(h)$ is the function one would use to find the value of h maximizing the area. You should not do the optimization in this case.)

Solution: The area of the garden in square feet is given by hw. In part (a), a formula for w in terms of h was found when h and w are the dimensions of a garden that will cost \$250 in supplies to construct. Thus, $A(h) = h\left(\frac{250-10h}{4h+5}\right)$.

Answer: $A(h) = h\left(\frac{250-10h}{4h+5}\right)$

c. [4 points] In the context of this problem, what is the domain of $A(h)$?

Answer: $0 < h < 25$