5. [12 points] Let $f(x)$ be a differentiable function defined for all real x with derivative

$$f'(x) = \left(e^{x-1}\right) x^4 (x+4)(x-3)^2.$$

a. [3 points] Find the x-coordinates of all critical points of $f(x)$.

Solution: Critical points of $f(x)$ occur where $f'(x)$ is zero or undefined. Since we are given a formula for $f'(x)$ that is defined everywhere, $f'(x)$ is defined for all real numbers x. So the only critical points of $f(x)$ occur where $f'(x) = 0$, i.e. at $x = 0, -4, 3$.

Answer: critical point(s) at $x =$ $0, -4, 3$

b. [6 points] Find the x-coordinates of all local extrema of $f(x)$. If there are none of a particular type, write NONE.
Justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Solution: All local extrema occur at critical points. Since f is continuous, we use the First Derivative test to classify the critical points from part (a) as local minima, local maxima, or neither.
We determine the sign of $f'(x)$ for $x < -4$, $-4 < x < 0$, $0 < x < 3$, and $x > 3$. On each of these intervals, e^{x-1}, x^4 and $(x-3)^2$ are positive. Thus, the sign of $f'(x)$ is determined by $(x+4)$, which is positive for $x > -4$ and negative for $x < -4$. Thus,

Interval	$x < -4$	$-4 < x < 0$	$0 < x < 3$	$x > 3$
Sign of $f'(x)$	-	+	+	+

By the First Derivative Test, $f(x)$ has a local minimum at $x = -4$. No other critical points are local extrema.

Answer: local min(s) at $x =$ -4

Answer: local max(es) at $x =$ NONE

c. [3 points] Suppose $f(1) = -7$. Use the tangent line approximation to $f(x)$ at $x = 1$ to estimate $f(1.1)$.

Solution: To find the tangent line approximation of $f(x)$ at $x = 1$, we first calculate

$$f'(1) = e^{(1-1)}(1^4)(1+4)(1-3)^2 = 20\,.$$

Thus, $f(1.1) \approx f(1) + f'(1)(1.1 - 1) = -7 + (20)(0.1) = -5\,.$

Answer: $f(1.1) \approx$ -5