6. [11 points] Consider the curve $\mathcal{C}$ defined by

$$e^{xy} = 4x - y^2 + 2\,.$$

a. [6 points] For this curve $\mathcal{C}$, find a formula for $\frac{dy}{dx}$ in terms of x and y.

Solution: Applying $\frac{d}{dx}$ to both sides of the equation for the curve, we have

$$e^{xy}\left(x\frac{dy}{dx} + y(1)\right) = 4 - 2y\frac{dy}{dx}\,.$$

Collecting all terms involving $\frac{dy}{dx}$ to the left hand side and factoring out $\frac{dy}{dx}$ gives

$$\frac{dy}{dx}\left(xe^{xy} + 2y\right) = 4 - ye^{xy}\,.$$

Thus, $\frac{dy}{dx} = \frac{4 - ye^{xy}}{xe^{xy} + 2y}$.

Answer: $\frac{dy}{dx} =$ $\frac{4 - ye^{xy}}{xe^{xy} + 2y}$

b. [2 points] Exactly one of the points below lies on the curve $\mathcal{C}$. Circle that one point.

$(2, 0)$ $\qquad$ $(1, -2)$ $\qquad$ $(1, 1)$ $\qquad$ $\boxed{(0, -1)}$

c. [3 points] Find an equation for the tangent line to the curve $\mathcal{C}$ at the point you chose in part (b).

Solution: The slope of the tangent line to $\mathcal{C}$ at (0,-1) is given by plugging $x = 0$ and $y = -1$ into the formula we found for $\frac{dy}{dx}$, which gives

$$\frac{4 - (-1)e^{(0)(-1)}}{0e^{(0)(-1)} + 2(-1)} = -\frac{5}{2}\,.$$

Thus, the tangent line is given by the equation

$$y = -1 - \frac{5}{2}(x - 0)\,.$$

Answer: $y =$ $-1 - \frac{5}{2}x$